

OPERATIONAL AND ECONOMICAL PERSPECTIVES OF CONSUMER RETURNS

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To Fazl

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ABSTRACT

**OPERATIONAL AND ECONOMICAL PERSPECTIVES ON
CONSUMER RETURNS**

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Customer return policies are one of common after sale services offered by a retailer in order to boost sales, improve customer satisfaction and diminish customer fit uncertainty. With such a service, the retailer accepts the return of a product after the sale has occurred, if it does not satisfy the customer's expectations. This study investigates consumer returns from three different perspectives. First, we start with a single period inventory planning problem of multi-variants in which customers have a right to return products in case of dissatisfaction. We assume that returns can be *as-good-as-new* condition after a minor restocking process and they are resalable in the same selling period. At the time of purchasing, a customer may choose to substitute her choice with another variant of the item, if the former is sold out. This, so called substitution, and resalable returns are important parameters in assortment planning that might affect the total profit dramatically. Under this setting, we aim to illustrate the effect of return and substitution on the optimal order quantities of variants and the total expected profit. We show that the total profit decreases as the probability of returns or the probability of resalable returns increases. In addition, the probability of substitution, or return, or

resalable return has a negative effect on inventory level of variants. Second, we analyze a retailer's return policy problem when the market consists of loss-averse customers who are more sensitive to losses than gains instead of being risk-neutral customers. We examine the situation in which a seller makes price and quantity decisions for a single item and also designs an appropriate returns policy in order to maximize his profit. We analyze the case where the seller offers either a full-refund or a partial-refund policy if he decides to accept returns or chooses not to accept any returns. With the full-refund policy, the seller reimburses the consumer the full price of the product if it does not fit the customer's preferences. With a partial-refund policy, the seller offers a refund which is strictly less than the purchase price. We assume that customers are strategic customers aiming to maximize their utilities of the product. With this model, we aim to analyze the impact of loss aversion on the seller's price and order quantity decisions. We show that the seller keeps fewer inventories as customers get more loss-averse and loss-aversion has a negative effect on the expected unit profit. Thus, the total profit decreases as loss-aversion. Finally, we present a model that investigates the effects of return policies on each of the two sellers' pricing decisions when these sellers engage in market size competition. Our model simultaneously addresses a consumer's purchase decision and the competing sellers' price decisions, along with their respective return policies. We assume that two competing sellers which do not have a capacity problem only decide their prices and return policies. The sellers may independently offer no-refund, full-refund of the price of the product, or a partial-refund. The market share of each party depends on his and the rival's price and return policy; customer valuations

of the product and the degree of competition between the sellers. Before purchasing, a consumer cannot evaluate the product's utility which is a decreasing function of the price set by the seller from which she chooses to purchase it and the disutility of purchase and/or return which is related to the physical distance between the consumer and the seller's location. The return policy of a seller may also affect a consumer's decision via her expected utility. Thus, pricing and return policy decisions play an important role in the division of the total market. We show that the full-refund policy yields a higher purchasing price compare to the no-refund or partial-refund policy. However, it only performs better in terms of profit when the product has a small salvage value and a partial-refund policy is not an option for the retailer. When the retailer offers the partial-refund policy, it always dominates the other refund policies. In addition, we show that the full-refund policy provides the highest consumer surplus since customers do not face any risk of misfit. On the other hand, the full-refund policy is socially efficient only when the salvage value is high. For low salvage values, the partial-refund policy yields a higher level of social welfare.

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Chapter 1

Introduction

It is a fact that a store's return policy is an important factor in a consumer's purchasing decision, as many products exhibit "personal fit uncertainty". For example, nowadays it is not uncommon at the retail level that a customer hesitates before buying a dress or a PC game since she is unsure if the dress will match her existing wardrobe or whether she will enjoy the PC game. Moreover, there exist cases such that a consumer purchasing a pair of shoes online is not happy with her purchase even it is in perfect condition, because she was unable to realize the exact attributes like size, color and material from its description. Many products, like shoes, kitchen appliances, game controllers and DVD players, need to be "experienced" by the consumers before they can be sure of their preferences. Such items are often referred to as "experience goods" (Che, 1996). The uncertainty about the benefits of this kind of a product decreases the willingness of a customer to purchase it, or may motivate a customer to search for a store offering a

lower price and extra services. In addition, new technologies and innovations lead to an increase in product variety, while marketing strategies create a feeling of “must-have” products. These phenomena together cause insatiable demand by consumers searching for the best overall deal. Thus, in order to boost sales, improve customer satisfaction and diminish customer fit uncertainty, retailers offer a number of after sale services. One such service is to accept the return of a product after the sale has occurred, if it does not satisfy the customer’s expectations. Customer return policies are pervasive in today’s retail business environment. The value of goods returned by buyers in the U.S. during 2009 exceeded \$180 billion, about 8% of total sales (National Retailer Federation, 2009). The main reason for returns is a mismatch between buyers’ expectations and actual experiences. It is contended that between 11% and 20% of all the electronic items purchased are returned, though only about 5% of them are truly defective (National Retailer Federation, 2009).

There are many types of return policies implemented in practice. For instance, policies such as exchange only, all sales final, store credit, money back and charging restocking fees, are all commonly used these days by retailers. A money back guarantee allows customers to return a product and receive a full price refund. By offering this return policy, a seller provides customers information about the value of the product. A customer who has a valuation greater than the purchase price will keep the product. Otherwise, the customer will return it. With such a guarantee, a retailer not only loses the potential revenue from returned products, but also may incur non-refundable shipping and handling costs that increase its total costs. A mitigating feature of a money

back policy is to charge a restocking fee to cover any processing and other related costs. In this case, the customer's risk due to product misfit and the seller's risk of excess inventory resulting from returned items are shared between the buyer and the seller. Other alternatives for a return policy is to offer store credit, or allow only a product exchange. Under the former policy, customers receive a credit equal to the full price of the product, when they return the product. An exchange only policy allows the buyer to replace the product with exactly the same type of product, e.g. an item of different color or size. Among these alternatives, we focus on the money back guarantee and return policies with a restocking fee throughout this study.

Return policies tend to protect customers against product misfits and increase customer satisfaction. Thus, such policies may stimulate purchases. Customers are more likely to buy products from a retailer with a generous return policy.¹ On the other hand, returns cause immediate operational consequences, which may have a negative effect on the retailer's revenues. They lead to an increase in inventories and handling costs. In addition, returned items are usually re-sold at discounted prices. In view of such trade-offs, establishing return policies can be a delicate task for a retailer. The retailer's aim is to balance the benefits and costs of returns. Thus, the retailer needs to carefully select the price, if it is not exogenous, while simultaneously design a return policy to attract customers and reduce their risks without resulting in excessive return related costs to the retailer. This problem has received comprehensive attention from both academic researchers and practitioners. This study analyzes consumers returns from three different

¹<http://business.time.com/2012/09/04/why-a-good-return-policy-is-so-important-for-retailers/>.

perspectives, aiming to make a contribution to the existing literature.

We start our analysis of consumer returns when a seller sells substitutable variants in a single inventory period and customers have a right to return products in the case of lack of satisfaction. We will derive the optimal order quantity of each variant which maximizes the seller's total expected profit considering the returns from customers. It is assumed that returned items can be brought to *as-good-as-new* condition after a minor operation such as testing and repackaging. Thus, they are resalable in the same selling period. In addition, when a customer does not find a particular variant that she is looking for, she may choose to settle for another variant. This, so called substitution, and resalable returns are important parameters in inventory planning, that might affect the total profit dramatically. In case of a return, the seller is exposed to some collection costs but there is no return fee for the customer. With this setting, we aim to illustrate the effect of return and switching on the optimal order quantities of variants and the total expected profit. Chapter 3, titled "The Effect of Substitution and Resalable Returns on Retailer's Inventory Decision", lists two major contributions to the literature: the effect of returns on optimal quantities and expected profit when they are multi-variants, and the effect of switching between products in the presence of returns.

In our first model, mentioned above, the seller decides on the optimal order quantity of each variant with exogenously given prices and constant return probabilities. Subsequently, we continue our analysis by relaxing both of those assumptions and modify our model to investigate consumer returns in a different environment. In the new model,

we assume that the seller sells a single unused product in a market which consists of loss-averse customers instead of risk-neutral customers. We remove the multi-variant assumption used in the previous model to eliminate the cross-effect of the substitution and reselling on the seller's decisions, so that we can solely focus on the impact of loss-aversion customers. The motivation of incorporating loss-averse customers is that consumers may sometimes be more sensitive to losses than gains, so they strongly prefer avoiding losses to acquiring gains. This phenomenon which is called *loss-aversion* is empirically well-established and a number of prior studies examine how such consumers behave when making decisions (Kahneman & Tversky (1979), Thaler (1980), Kahneman et al. (1991)). In the current model, the seller tries to find the optimal price and order quantity which maximizes his total profit. In addition, he designs an appropriate returns policy. Instead of offering just a money-back guarantee, the seller may choose to charge a restocking fee for returns or choose not to accept any returns. We will call the money-back guarantee as a full-refund policy and the return policy with a restocking fee as a partial-refund policy. We model return policies with a variable which is equal to the refund that a customer receives when she makes a return. With the full-refund (or no-refund) policy, the seller reimburses the consumer the full price of the product, (or nothing) if it does not fit the customer's preferences. Thus, the refund is equal to the price of the product (or zero). With a partial-refund policy, the seller offers a refund which is strictly less than the purchase price. The difference between the full price and the refund amount may be termed a restocking fee or a non-refundable charge that the seller imposes on consumers. The seller's policy about returns and pric-

ing directly influence consumers' purchasing decisions. Each consumer who purchases at most one product and faces an uncertainty in product valuation before she experiences the product. Like the seller, a consumer aims to maximize her utility of the product in question. Chapter 4, titled "Pricing and Customer Returns Policies with Loss Averse Customers", aims to analyze the impact of loss aversion on the seller's price and order quantity decisions under different return policies.

Next, we will focus on the effect on competition on the seller's pricing decisions when each seller chooses his return policy independently. To explore the effect of competition, we present a model where two sellers engage in market size competition. The main motivation of analyzing the competition effect is that most consumer returns studies in the literature focus on monopolistic sellers. In Chapter 5, labeled "Pricing and Customer Returns Policies for Two Competitive Retailers", we will attempt to fill the gap resulting from the impact of competition and return policies on sellers' pricing decisions. Our modified model simultaneously addresses a consumer's purchasing decision and the competitive sellers' pricing decisions, along with their respective return policies. In this model, we assume that two competitive sellers which do not have capacity restrictions only decide their prices and return policies. These decisions are made independently before the selling season. Each seller chooses to offer one of three different refund options which determine his return policies. The seller may offer no-refund, a full-refund or a partial-refund. The market share of each seller depends on his and the rival's selling price and return policy, customer valuations of the product and the degree of competition between the sellers. Before purchasing, a consumer cannot evaluate the product's

utility which is a decreasing function of the price set by the seller from which she chooses to purchase the product and the disutility, represented by the physical distance between herself and the seller's. The return policy of a seller also affects a consumer's decision via her expected utility. Thus, pricing and return policy decisions are likely to play an important role in the division of the total market and sellers' total profits.

The models proposed in this study show that benefits of return policies depend on several parameters such as the salvage value, customers' behavior and valuations, and the competition level. We observe that the return policies improve as the product has a good salvage value in a monopolistic case. In a competitive environment, the effect of the salvage value is positive on the return policy except some special cases such as when the retailer offers the no-refund policy and his rival offers a partial-refund policy. From the retailer's perspective, a properly designed partial-refund policy performs better than the full-refund and no-refund policies with respect to the total profit. On the other hand, the full-refund is always preferable by customers and it always provides the highest consumer surplus. In addition, when we consider the effect of return policies on social welfare, we observe that the partial-refund policy is socially efficient when the retailer has a lower salvage value, while the full-refund policy yields a higher level of social welfare when the salvage value is high.

The key contribution of this dissertation is to shed some light on consumer return policies, while filling some of the existing gaps in the literature of consumer returns. As alluded to earlier, we propose three models to cover different market scenarios. We

start our analysis with a basic model involving a single retailer who maximizes his total revenue from substitutable product variants. The key contribution of this model is to explore the effects of switching between products in the presence of consumer returns. In the second model, we focus on loss-averse customers and analyze their effect along with return policies, on a retailer's price and quantity decisions for a single item. This model helps us understand that loss averse customers should not be treated like risk neutral consumers, since both types show different purchasing behavior and ignoring the phenomenon of loss averse consumers may adversely affect the seller's profit. Finally, the third model contributes to the consumer return literature by analyzing return policies in a competitive environment. These models are suitable for different scenarios of the market environments. In addition, although we assume that the retailers sell a homogenous product but are differentiated in terms of their service and other characteristics; our model is still valid when the sellers sell substitute products, where consumers have different valuations for such products.

The remainder of the study is organized as follows. Chapter 2 provides a general review of the literature concerning consumer returns. Following this, the above mention models and their analysis are focused in the next three chapters. Chapter 3 discusses the basic model, which investigates the effect of substitution and resalable returns on seller's inventory decision. Chapter 4 presents seller's pricing and returns policy decisions with loss-averse customers and in Chapter 5, we discuss the effect of consumer returns on pricing decisions of two competitive sellers'. Finally, Chapter 6 summarizes our findings from the three proposed models and Appendix provides the proofs of all the technical

results.

Chapter 2

Literature Review

This chapter first presents a review of two different streams of the extent literature, which provide an overall view of the concepts that will be utilized in the subsequent chapters of this dissertation. These two streams deal with strategic consumer behavior and the dynamics of consumer returns. We then provide the additional literature directly related to each of our models.

Several aspects of consumer behavior have received considerable research attention in recent years. A comprehensive review of the literature on consumer behavior modeling can be found in Shen & Su (2007). We outline below a number of related studies that provide a broad view of this area of study. There are different streams of research on consumer behavior. For example, Su (2007), Elmaghraby et al. (2008) and Aviv & Pazgal (2008) consider dynamic pricing problems where customers make their purchasing decisions depending on the expected future price. Aviv & Pazgal (2008) is one of the

earliest papers which investigate the aforementioned problem. They outline a model where the seller reduces its price at a fixed point in time. The authors explore the seller's behavior under two discounting policies. Under the first policy, the seller chooses a discount price which depends on the left-over inventory, while under the second one, the seller chooses the discount price at the beginning of the selling season. Following this study, Elmaghraby et al. (2008) consider a multi-period pricing problem in which the seller mark down the original price more than once. Su (2007) studies the impact of strategic customer, where the seller has the option to increase or decrease the price over time. Another group of studies investigates the sellers' inventory decisions when strategic consumers wait for markdowns, which usually occur at the end of the selling season due to an excess inventories. For example, Su & Zhang (2008) study the implications of this type of consumers on supply chain performance. They observe that a high inventory level decreases the regular season demand by increasing the chance of having leftover units. Thus, a high inventory level increases the consumers' tendency of buying the product on sale at the end of the selling season. Furthermore, Cachon & Swinney (2009) demonstrate that the value of quick response strategies, which enables the seller to procure additional inventory after demand information is updated, is much higher in the presence of strategic consumers. In another paper, Jerath et al. (2010) discuss the direct last-minute sales and opaque sales. They demonstrate that opaque selling enhances the seller's profit when customer valuations are low.

Liu & van Ryzin (2008) consider capacity rationing in a two-period model in which the seller pre-commits to prices in both periods. These authors observe that the effec-

tiveness of rationing depends on consumer risk preferences and show that it is never optimal to ration risk-neutral customers. Following this, Liu & van Ryzin (2011) consider a case where consumers learn about a seller's capacity using a moving average smoothing process, and decide whether or not to wait for a possible markdown. In contrast to the previous paper, the authors demonstrate that the optimal policy may converge to a rationing equilibrium. In addition, Dana & Petruzzi (2001) consider a newsvendor model by assuming that there exist expected utility maximizing consumers and the seller faces endogenous demand depending on price and inventory level and consumer. The authors show that the seller holds more inventories, provides a higher fill rate, attracts more customers, and earns higher profits by internalizing the effect of its inventory on demand. In another paper, Su & Zhang (2009) also study the effect of inventory information on consumer purchase. They find that the seller can improve profits by making a combination of inventory commitment and availability guarantees. Next, Yin et al. (2009) focus on the impact of product availability and inventory level information on strategic consumer behavior. They demonstrate that creating a sense of shortage risk can increase profits.

A basic common assumption in studies mentioned above is that consumer demand is endogenous and depends on the seller's inventory and pricing decisions. Under this assumption, these studies consider two different types of consumer's risk: first, the risk resulting from concerns about a decrease in a product's price in the future; and secondly, the risk caused by concerns about its availability. In this work, we investigate consumers' risk due to product misfit. In other words, consumers are uncertain about

their valuations, so they may face a potential product misfit if their valuations turn out to be low. In such circumstances, return policies can encourage consumers to purchase the product in spite of these concerns.

There is a growing body of literature on consumer return policies. Davis et al. (1995) propose a model with a money back guarantee assuming that consumer valuation after a purchase is a Bernoulli random variable. This paper contends that a full refund policy is more profitable than selling a product without a refund option, when the seller's salvage value is greater than the transaction costs resulting from a consumer return. Che (1996) models a money back guarantee policy while allowing customer valuations to follow a general distribution. He shows that a money back guarantee is preferable to a no-refund policy when consumers are highly risk averse, or when the retail price is relatively high. Furthermore, Chu et al. (1998), Su (2009) and Xiao et al. (2010) analyze partial return offers. Chu et al. (1998) assume that customers receive a portion of the purchase price of the product and incur a cost to return it, whereas Su (2009) and Xiao et al. (2010) consider a return refund less than the price. In addition, Davis et al. (1998) consider partial refunds in conjunction with impediments (e.g. requirement of receipt, original box or packaging, or a time limit) and study the effect of such obstacles on consumer return decisions. Among these studies, Davis et al. (1995), Che (1996) and Chu et al. (1998) focus on the case with fixed capacity and determine an optimal price to prevent potential returns, whereas Su (2009) and Xiao et al. (2010) focus on both inventory and pricing decisions simultaneously.

A common assumption in the consumer return literature involves homogeneous customers. Davis et al. (1995), Che (1996), Davis et al. (1998), Su (2009) and Xiao et al. (2010) examine *ex ante* homogeneous customers who face valuation uncertainty before the purchase. Liu & Xiao (2008), on the other hand, consider heterogeneous customers who differ in their valuations before the purchase and analyze a single return policy that serves a fraction of the population, a menu of returns that serves the entire population and an inventory rationing policy. They show that when the inventory level is high, or the procurement cost is low, offering a menu of returns which serves all customers is more desirable for the seller than offering a single return policy, which serves high-end customers, or an inventory rationing policy. Matthew & Persico (2007) and Shulman et al. (2009) also consider customers exposed to two sources of uncertainty: individual valuation of the utility of the product and the product's fit with consumer preferences. These papers assume that consumers know their valuations of the product before purchase. Matthew & Persico (2007) show that a retailer selling a single product benefits if consumers are uninformed about the product value with the promise of a large refund, greater than the salvage value. In contrast, Shulman et al. (2009) consider a retailer offering two horizontally differentiated products and allow customers, who incur both a "hassle" or inconvenience cost and a restocking fee, for exchanging or returning the unsuitable product. In this paper, it is shown that the restocking fee is used both to lessen the cost of returns and to alter consumer behavior. In addition, these authors prove that the exchange opportunity increases the customer's willingness to pay more for the product, thus allowing an increase in the product's price.

There are several other papers that present different functions of consumer returns policies. For example, Moorthy & Srinivasan (1995) show that attractive return policies signal high product quality, when both the retailer and the consumers are subjected to large transaction costs. Heiman et al. (2002) model money-back guarantees as put options. Furthermore, Hess et al. (1996) develop a model by imposing non-refundable charges for mail-order catalogs and show that inappropriate returns can be controlled via non-refundable charges, which increase with the value of the order.

Another stream of customer returns focuses on the effect of returns in closed-loop supply chains. These studies investigate policies for managing inventories when there are customer returns. Fleischmann et al. (2002) and DeCroix et al. (2005) provide examples of this line of inquiry. The returned products in these studies are either remanufactured and sold as new or disassembled, so that their components can be reused. In these studies, the price is taken as an exogenous parameter, thus pricing decisions are not considered. A common assumption of these studies is that the rate of product returns is a known fraction of past sales. Fleischmann et al. (1997) provide an extensive review on the reverse logistics and remanufacturing literature.

In the following sections, we review the literature directly related to each of the suggested models.

2.1. The Effect of Substitution and Resalable Returns on Retailer's Inventory Decision

Inventory management of multiple products has been studied extensively in the operations management literature. The associated models subject to various constraints are usually solved using Lagrangian multipliers or heuristic approximations. Inventory models for multiple products, which consider stockout based substitution, focus on the stocking decisions, given a selection of products. In this chapter, we will summarize multi-item inventory problems in which there is substitution between products. Some of the earliest works which consider substitutable products are McGillivray & Silver (1978), Parlar & Goyal (1984) and Pasternack & Drezner (1991). McGillivray & Silver (1978) use simulation and develop a heuristic model to determine the optimal order quantities for the two products. They show that substitution can reduce the total holding and shortage cost. Parlar & Goyal (1984) study an inventory model for two partially substitutable products. Next, Pasternack & Drezner (1991) use the same model for the full substitution case, where higher quality product can substitute low quality level product.

Most of the literature related to returns consider the management of return flows which includes all logistic activities to collect, disassemble and reimburse used products to increase its salvage value. These activities are generally known as reverse logistics. The studies in this context generally consider cases in which returns need extensive repair before reselling or recycling them. Two examples from recent studies in this area, which are similar to this study are Vlachos & Dekker (2003) and Mostard & Teunter (2006).

Vlachos & Dekker (2003) discuss the newsboy problem with returns. They assume that products can be resold only once and a fixed percentage of sold products is returned and is resalable. In a subsequent study, Mostard & Teunter (2006) discuss the case in which products can be resold several times during the same period.

2.2. Pricing and Customer Returns Policies with Loss Averse Customers

A common assumption in the consumer return literature involves risk-neutral customers. Studies listed in Chapter 2 examine risk-neutral customers who have zero utility if the true valuation of the product turns out to be lower than the purchase price. However, it is well known that in many situations customers are motivated by avoiding a loss than acquiring a gain. Kahneman & Tversky (1979) is the first to study individuals's evaluation of potential losses and gains. Their paper models how people make choices when they face alternatives that involve risk. From this empirical study, it is found that gains are evaluated differently from losses and outcomes with certainty are preferred relative to uncertain outcomes. Thaler (1980) consider the same idea presented by Kahneman & Tversky (1979) for decisions involving riskless choices. He observes that people often are more willing to give up an object than paying to acquire it. In other words, people put a higher value on objects that they own those they do not. Thaler (1980) labels this behavioral discrepancy as the *endowment effect*, since the value of the object

changes once it is incorporated in one's endowment. The term *Loss-aversion* was first used by Kahneman & Tversky (1984) to label the phenomenon that the disutility of giving up an object is greater than the utility associated with acquiring it.

Loss-aversion is the one of the characteristics of consumer preference which is observed in a variety of experimental situations such as monetary gambles and risky or riskless decisions. In this broad literature, however, the effects of such behavior on operational decisions. There are relatively few studies which examine the effect of loss-aversion on pricing . Popescu & Wu (2007) study the effect of loss-aversion on dynamic pricing strategies. Heidhues & Köszegi (2005) investigate the monopolist's pricing strategy of a retailer selling to loss-averse consumers, then extend their model to accommodate price competition with differentiated products (Heidhues and Köszegi, 2008).

In this model, we attempt to fill the research gap that exist in evaluating the impact of loss-averse consumers on a seller's inventory and pricing decisions. We extend the classical newsvendor model where a seller faces random market demand. We assume that the seller faces loss-averse consumers and determine whether to offer a return policy to consumers whose true valuations turn out to be low after experiencing a product.

2.3. Pricing and Customer Returns Policies for Two Competitive Retailers

Our work is closely related to three research streams in the existing body of literature: strategic consumer behavior, consumer returns policies and price competition between sellers. The literature pertaining to strategic customer behavior and consumer returns are provided in Chapter 2. Here, we will summarize the literature of price competition between sellers.

In this model, we explore how the return policies affect the competitive sellers' profits, considering first the effects of a full-refund policy and then those of partial-refund policies.

We use Hotelling's conceptual framework (Hotelling 1929) to model the competitive environment. Hotelling (1929) introduces a model of spatial competition where two sellers are located at the two end points of a market offering homogeneous products. In this model, consumers who are uniformly distributed on the straight line have unit demands and each of them buys exactly one product from the seller that offers the larger expected utility based on price and travel distance. In more recent related research, a number of Hotelling's assumptions have been relaxed. Such studies (Kim & Serfes 2006, Sajeesh & Raju 2010, Pun & Heese 2010) use Hotelling's framework for modeling positioning and pricing decisions in a market in which consumers seek variety and are allowed to make multiple purchases. Jerath et al. (2010) also consider Hotelling's model

in a scenario where the two competing sellers make their last-minute sales through an opaque intermediary.

It is to be noted that the studies cited in Chapter 2 on consumer return policies are largely focused on monopolistic cases. Research that simultaneously deals with return policies and competition is still at a stage of infancy. Shulman et al. (2011) is one of such studies which optimize price and restocking fee decisions of a duopoly selling horizontally differentiated product for heterogeneous customers. One major finding here is that competing sellers who sell differentiated products charge a higher restocking fee than a monopolist to keep consumers from exchanging for the competitor's product and to generate an additional source of revenue from returns. In addition, sellers charge higher restocking fees when consumers are less-informed about a product's match. The difference between our model and Shulman et al. (2011)'s model is that it assumes horizontally differentiated products and consumer heterogeneity, while we assume both sellers sell the same product to homogeneous consumers.

Guo (2009) is another study which investigates the profitability of partial refund policies in a competitive market. This work assumes two *ex ante* identical service-selling retailers with capacity constraints which may choose to offer refunds for canceling advance purchases. This work shows that when the aggregate capacity of the sellers is relatively small, both sellers should adopt partial refund policies. In contrast, when the total capacity is relatively large, they should both follow no refund policies. Furthermore, it is shown that offering partial refunds increases competition and lowers the sellers'

profits compared to offering zero refunds. The difference between our model and Guo's model is that Guo focuses on a service market and assumes that canceled advance purchases can be resold. In our model, we focus on product returns, and allow sellers to salvage the returned products.

Chapter 3

The Effect of Substitution and Resalable Returns on Retailer's Inventory Decision

Evolving technologies, global competition, and sophisticated customers lead to an increase in product variety in most industries such as manufacturing automobiles, electronic devices and consumer apparel. However, holding inventory for each kind of product may not be possible for a retailer due to budget and space constraints. In addition, an increase in product variety does not guarantee an increase in long run profits. In fact, increased variety may have a negative impact on the retailer's profit, since it may result in high production cost, excess inventories or lost sales. Thus, a proper assortment and inventory planning are crucial operational issues for retailers. On the other hand, an

increase in competition and customers' awareness about their rights force retailers to offer some after sale protections. One of these protections is to accept the return of the product after sale, if it does not meet a customer's expectation. In Chapter 1, we have discussed the importance of a return policy and how it may affect the retailer's profit. Offering a return policy, along with an inventory planning can serve as an important operational, strategic and marketing tool for the retailer. Assortment planning involves a selection of products that the retailer includes in a manufacturing system or in a market, whereas inventory management is concerned with how many of each item should be manufactured or ordered. It is important that retailers manage this process efficiently since poor assortment and inventory planning may cause surplus inventory and/or lost sales. While making inventory decisions, a return policy should be taken into account, since the presence of return flows may have a high impact on inventory levels, especially if returned items are resalable in the primary market as new products. In the retailing business, consumer returns policies and inventory planning are usually considered as separate problems and decisions in each area are made independently (Stock et al. (2006)). In this study, we will examine the retailer's inventory and return policy decisions simultaneously.

While considering the assortment planning problem, note it is seen that if a retailer reduces variety in all item categories, the store becomes less attractive, as some customers are likely to defect to competing retailers, which causes it to lose profit. If a store has an unlimited budget and storage capacity, it can order as much variety as possible. However, in reality this is not the case in reality. Thus, assortment planning becomes popular as

an increase in product variety. In addition, retailers expand their assortment of products in terms of variety and depth to increase their market share as markets become more competitive. Higher variety usually leads to higher sales by increasing retailer's total demand, but this comes with higher operational complexity and cost. The other effect of assortment planning is the effect of substitution. In practice, customers may prefer switching from one product to another one when the desired product is out-of-stock and if the other product differs slightly from the former in terms of color, style or functions. The impact of each items demand on another's demand is called the substitution effect. The existence of substitution significantly alters the retailer's merchandizing decisions, i.e. choosing product assortments, since in such a case an unsatisfied demand may not be simply lost, but may be an additional demand for another product.

Earlier in Chapter 1, we have emphasized the importance of returns in retailing. The management of return flows has become more important in the past decade, since the financial impact of a return policy may be substantial for a retailer. For instance, Stock et al. (2006) indicate that reverse logistics costs account for 0.5% of the total United States Gross Domestic Product and the average return rate for online retail sales is 5.6%, although this may vary by product and time of year. These statics show that a high volume of returns may result in a significant increase in operational costs. For instance, for an internet based retailer, the cost of processing a return can be two to three times of that of an outbound shipment. Selling returns in a secondary market or directly salvaging them are some traditional methods. On the other hand, they can be resold in the primary market after some recovery process. In the latter case, inventory decisions

are significantly affected by returns. These decisions are more challenging when the retailer sells more than one item under product substitutability. As the product variety increases, the complexity of processes used to manage returns become more critical. Thus, returns become more costly for the retailer. In this chapter, we explore how inventory decisions of each variant changes when returns are taken into account. We assume that returned products are commercial returns such that after a minor operation, e.g. repackaging, they can be sold *as-good-as-new* with the same price as that of the original product.

In our first model, we consider the inventory planning problem of n previously chosen product variants. We assume that the retailer's problem is to choose the appropriate inventory level of each variant when consumers have the option of substituting their first choice if it is not in the stock. In addition, consumers are allowed to return a product if it does not meet with their expectations. We assume that returns can be treated as *as-good-as-new* after a minor operations. Thus, they are resalable in the same selling period after testing and repackaging. The retailer places one order for each product and these orders arrive before the selling period begins. If there is a shortage of a specific product, the customer has two options. She can either prefer not to purchase her preference or switch to another product. If she prefers switching, then she purchases a product which is less preferable among all other available products. Thus, it is assumed that different types of products can be ranked based on customers preference. Our assumptions are that holding and ordering or set-up costs are negligible. In addition, the retailer is assumed to have not storage or shelf space constraint. In the case of a return, the

retailer is exposed to some collection costs but there is no return fee for the customer. With this setting, we aim to illustrate the effect of returns and switching on the optimal order quantities of product variants and the total expected profit.

In the next section, we introduce the model for this scenario with the underlying assumptions and notations. This section also includes the optimal quantities of each product type. Then in Section 3.2, a numerical example is presented for the case with two products. Finally, Section 3.3 summarizes the basic results found with the proposed model.

3.1. Model Development

We assume that there are n product variants and the retailer chooses the order quantity for each item. There is a single replenishment opportunity and orders arrive before the start of the selling season. Over the selling season, the gross demand for product variant i is G_i with a mean of μ_i and standard deviation of σ_i . It is assumed that the distribution function of demand for each variant is independent. This provides us more explicit structural results. On the other hand, the effect of substitution eventually creates a dependency between product demands. It is assumed that the realization of demand occur at the beginning of the period and the original demand of each product is satisfied first. Then, if there is excess inventory of one product and excess demand for another, some or all of the excess demand for the latter may be satisfied from the stock of the former according to a probabilistic substitution behavior.

Customers are allowed to return the product within a certain time limit. If a product is returned, the customer gets a full refund. Each returned product results in a cost of d independent of the variants, whereas there is no cost or restocking fee incurred by the customer. We assume that each product sold is returned with the same probability of β . Any returned product can be resold after it is inspected and put back on the shelf before the selling season is over. Returned products are assumed to be as-good-as new and they can be sold at the same price as new products if there is sufficient demand for them. A returned product is resalable with a fixed and known probability of k . We assume that the average time between a sale and a return, including its recovery time is small enough to place the recovered return item on the shelf during the current season.

We assume that the product variants can be ranked in terms of popularity. The most preferable variant is denoted by a subscript n and the least preferable variant is denoted by a subscript 1. In the case of a shortage of variant i , a customer may prefer to buy a substitute of this variant. We assume that if a customer prefers to substitute her request in case of a stockout, she substitutes it with any of the variants which have a rank lower than her preferred variant. The substitution probability α_{ij} $i = 1, \dots, n$, $j = 1, \dots, i - 1$ refers to the fraction of switching from variant i to variant j and α_{ii} indicates the customer is not willing to substitute her choice with any other variant. In the latter case, each demand not satisfied results in a shortage cost v_i . Lastly, any unsold product at the end of the sales period has a salvage value s .

The objective of the retailer is to determine the order size q_i , $i = 1, \dots, n$ of each

variant in order to maximize the total expected profit. The retailer pays a cost c_i per unit for an order of q_i units and receives a sales price p_i per unit sold. Since the retailer needs to consider resalable returns while finding the optimal order quantities, it is better to consider the net demand of variants instead of gross demand. The net demand of variant i , N_i is equal to the total number of demanded products that are either not returned or returned but not resalable plus the demand caused by switching from higher level variants. When a demand for a variant i occurs, the retailer receives the sales revenue p_i for each product not returned. Thus, the per unit expected revenue for these products is $(1 - \beta)p_i$. If the product is returned, the retailer pays a per unit expected collection cost which is equal to βd . If the returned product is not resalable then it results in a per unit expected salvage value of $\beta(1 - k)v$. Thus, the unit expected revenue of a unit of gross demand is to $p_i^G = (1 - r)p_i - \beta d + \beta(1 - k)s$. By using this, we can derive the expected revenue of net demand.

Assume any product can be sold repeatedly until it is either not returned or not resalable. This assumption is reasonable when the average time between a sale and a return plus the recovery of the returned item is small relative to the length of the selling season, e.g. mail order retailer for a seasonal product (Mostard & Teunter (2006)). The unit expected revenue of satisfying a net demand includes selling the product until it is either not returned or returned but not resalable, when it is salvaged. Mathematically, the unit expected revenue of satisfying a net demand of variant i is $p_i^N = p_i^G[1 + \beta k + (\beta k)^2 + \dots] = p_i^G/(1 - \beta k)$. The ratio $1/(1 - \beta k)$ can be interpreted as the probability that product is not returned or is not resalable. The expected revenue p_i^N should be

larger than c_i , otherwise, it would result in an expected loss. Similar to the unit expected revenue, the expected net shortage cost of not satisfying a net demand is $v_i^N = v_i/(1 - \beta k)$.

The net demand, N_i , which is the total number of demanded products that are either not returned or returned but not resalable plus the demand caused by switching from higher level variants has a mean μ_i^N and variance $(\sigma_i^N)^2$. These values can be calculated from the mean and standard deviation of gross demand as follows (Mostard & Teunter (2006)):

$$\begin{aligned}\mu_i^N &= (1 - \beta k)\mu_i \\ (\sigma_i^N)^2 &= (1 - \beta k)^2\sigma_i^2 + \beta k(1 - \beta k)\mu_i\end{aligned}$$

Since the distribution function of each variant's demand is assumed to be independent, our problem is maximization of total expected profit (TEP) which can be represented as a summation of individual expected profits, $EP(q_i)$ as follows:

$$TEP(q_1, \dots, q_n) = \max_{q_1, \dots, q_n} \sum_{i=1}^n EP(q_i) \quad (3.1)$$

where the expected profit of variant i is:

$$\begin{aligned}EP(q_i) &= p_i^N \{ \mu_i^N + \sum_{j=i+1}^n \alpha_{ji} E[(N_j - q_j)^+] - \alpha_{ii} E[(N_i - q_i)^+] \} - c_i q_i \\ &\quad - v_i^N \alpha_{ii} E[(N_i - q_i)^+] + s \{ q_i - (\mu_i^N + \sum_{j=i+1}^n \alpha_{ji} E[(N_j - q_j)^+] \\ &\quad - \alpha_{ii} E[(N_i - q_i)^+]) \} \end{aligned} \quad (3.2)$$

where $x^+ = \max(x, 0)$ so $E[(N_j - q_j)^+]$ represents the expected net shortage of variant j . The retailer receives the sales price p_i^N for each unit of variant i which are not returned. The expected net demand for variant i includes the following three terms: the mean net demand of variant i , μ_i^N , the expected total demand due to substitution, $\sum_{j=i+1}^n \alpha_{ji} E[(N_j - q_j)^+]$, and the expected shortage resulting from customers who do not want to switch their preferences, $\alpha_{ii} E[(N_i - q_i)^+]$. Since we assume only downward substitution, variety types, which have ranks j larger than $i+1$, can increase the demand of i . The expected total increase in demand by any larger variant type j is the expected stockout of variant j times the probability of substitution between these types. In addition, any unsatisfied demand for variant i is substitutable with the variant having a lower rank. Thus, the net shortage for variant i is equal to the fraction of unfulfilled demand which is not substituted with lower ranked variants. Recall that the expected net revenue p_i^N includes the salvage revenue if a product is returned but not resalable. Thus, the salvage value in equation 3.2 is the remaining salvage revenues from left over inventory.

By observing the expected profit of each variant, $EP(q_i)$, it can be seen that $EP(q_i)$'s are not independent from each other. Each variant having larger rank than variant i leads to an additional demand for variant i . However, since the demand distribution function of each variant is assumed to be independent, the TEP can be separated into n subproblems. The total expected profit TEP is the summation of individual expected

profit functions $TP(q_i)$ which is stated as follows:

$$\begin{aligned}
TP(q_i) = & p_i^N \{\mu_i^N - \alpha_{ii} E[(N_i - q_i)^+]\} - c_i q_i - v_i^N \alpha_{ii} E[(N_i - q_i)^+] \\
& + s \{q_i - (\mu_i^N - \alpha_{ii} E[(N_i - q_i)^+])\} + \sum_{j=1}^{i-1} p_j^N \alpha_{ij} E[(N_i - q_i)^+]. \quad (3.3)
\end{aligned}$$

$TP(q_i)$ includes the expected profit from satisfying demand of variant i with the available inventory of variant i , satisfying the shortage of variant i from lower ranked variants, the expected shortage cost of unmet demand of variant i and salvage value of excess demand of variant i . The optimal order quantity of variant i that maximizes the expected profit in equation 3.3 is equivalent to minimizing

$$E[(N_i - q_i)^+] \{\alpha_{ii}(p_i^N + v_i^N - s) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij}\} + (c_i - s)q_i. \quad (3.4)$$

The first order optimality condition from 3.4 leads to the following optimal order quantity for variant i :

$$q_i^* = F_i^{-1} \left(\frac{\alpha_{ii}(p_i^N + v_i^N - s) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij} - c_i + s}{\alpha_{ii}(p_i^N + v_i^N - s) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij}} \right) \quad (3.5)$$

where F_i^{-1} is the inverse cumulative distribution function of variant i 's net demand. These values are global optimal values since the function in equation in 3.3 is strictly concave with respect to q_i . The structure of the optimal ordering policy is similar to the classical newsvendor ratio, the difference being that the distribution function and cost parameters are written in terms of the net demand and there are substitution effects

between variants.

The challenging part of this problem is to estimate the distribution function of net demand. In practice, it is almost impossible to estimate the entire distribution function. However, Mostard & Teunter (2006) show that the gross demand function is almost equal to the net demand function. So, it is reasonable to assume net demand follows the same type of distribution as gross demand. For instance, if the gross demand is normally distributed then the net demand can be assumed to be normal.

3.2. Computational Study

In this section, we will compare the effects of returns and substitution on the optimal order quantity and the optimal expected profit. For simplicity, we will consider the case with two variants: variant 1 is the lower rank variant and variant 2 is the upper rank. Only variant 2 is substitutable. Both variants' demand functions are normal with means and standard deviations of (150, 15) and (400, 40), respectively. Both variants have equal coefficient of variance to eliminate the effect of variance on the decision variables. The other relevant parameters are shown in Table 3.1.

To capture the effect of varying the return proportion, we use four different values: $\beta \in \{0, 0.1, 0.3, 0.5\}$. We assume that the probability of returned products which are resalable is in the set of $k \in \{0, 0.4, 0.8\}$. Finally, to show the impact of substitution, we use the set of $\alpha_{21} \in \{0.1, 0.5, 0.7\}$. Recall $\alpha_{11} = 1$ and $\alpha_{22} = 1 - \alpha_{21}$.

First, we will observe the effect of β and k on the unit expected revenue of satisfying

Table 3.1: Test Parameters
Variant 1 Variant 2

p_i	100	300
c_i	20	20
s	8	8
d	10	10
v	30	30

a net demand, $p_i^N = p_i^G / (1 - \beta k)$. From the first order derivative, it can be seen that the unit expected revenue is a monotonically decreasing function of the return probability, β . Whereas, the effect of the probability that a returned product is resalable, k is not monotonic (see in Appendix). Depending on the probability of return, β and the collection cost, d , the relation between p_i^N and k may be positive or negative. However, unless the return probability or the collection cost d is too large, p_i^N is an increasing function of k .

Table 3.2 shows the results of our computational exercise conducted with Excel. Figure 3.1, 3.2 and 3.3 summarize graphically some of the results provided in Table 3.2. Figure 3.1 shows the optimal quantity of each variant for $k = 0.5$ and $\alpha_{21} = 0.5$. It is seen that the individual effect of expected return probability on the optimal quantity is negative. Recall that the price of net demand decreases with the expected return probability. The expected net demand also decreases with the return probability. Thus, it is expected that the optimal order quantity of each product decreases as the return probability increases for a given switching and resalable product probabilities. In addition, the optimal total profit decreases as a result of decreasing price and quantity.

Figure 3.2 shows the optimal quantity of each variant when $\beta = 0.3$ and $\alpha_{21} = 0.5$. For return probabilities that are not extremely large, the effect of resalable product probability has a positive effect on the net price. Although, the net price rises with k , the optimal order quantity for each product decreases when the probability of resalable product increases. Recall that when either k or β increases, the mean net demand decreases, whereas the expected demand that can be satisfied with resalable returns increases. An increase in either of these probabilities has a positive effect on the expected shortage, i.e. the expected shortage decreases as either probability increases. Thus, the retailer orders smaller quantities, since he has an option to use the returned items. In addition, the optimal profit from each variant decreases due to the quantity decrease.

Finally, Figure 3.3 shows the effect of the switching probability on the optimal order quantity. Since, only variant 2 can be substituted with variant 1, the optimal order quantity of variant 1 is not affected by the change in the switching probability. The optimal order quantity of variant 2, on the other hand, decreases as the probability of the customer's willingness to switch their unmet demand increases. Again, the retailer uses the opportunity to replace the unsatisfied demand of variant 2 with variant 1. Thus, the effect of substitution on variant 2's is negative.

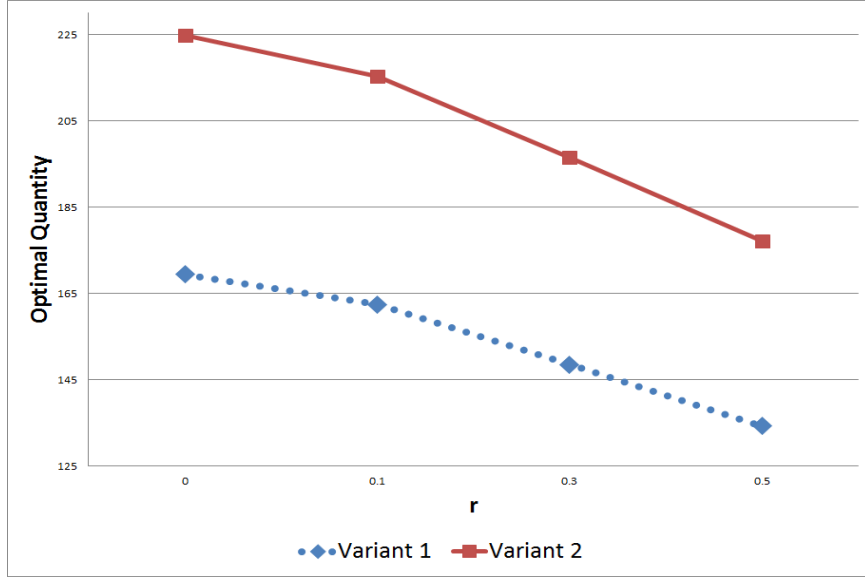


Figure 3.1: Optimal quantities vs β , $k = 0.4$, $\alpha_{21} = 0.5$

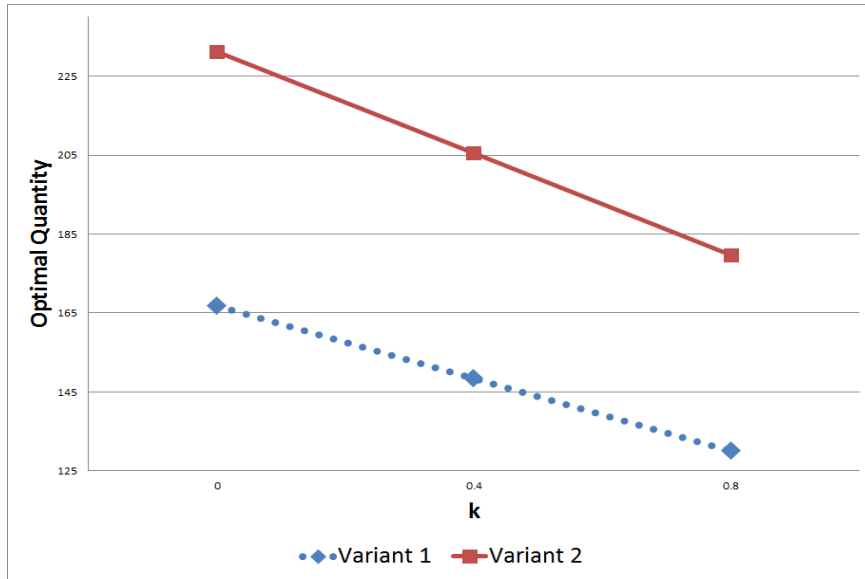


Figure 3.2: Optimal quantities vs k , $\beta = 0.3$, $\alpha_{21} = 0.5$

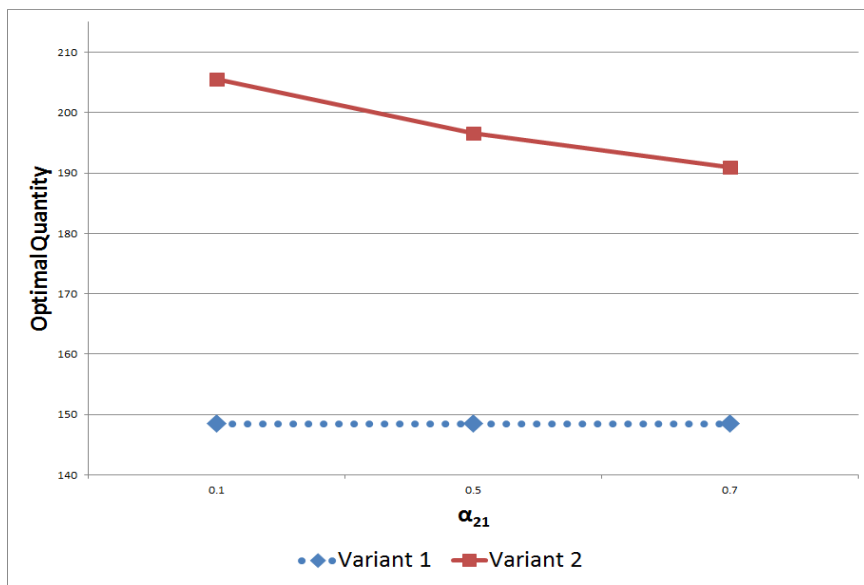


Figure 3.3: Optimal quantities vs α_{21} , $\beta = 0.3$, $k = 0.4$

Table 3.2: Numerical results with two products

$k = 0$												
$\alpha_{21} = 0.1$				$\alpha_{21} = 0.5$				$\alpha_{21} = 0.7$				
$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0.7$
q_1	169.4	168.6	164.4	169.4	168.6	166.8	164.4	169.4	168.6	166.8	164.4	164.4
q_2	234.4	233.4	231.2	224.7	223.6	220.9	217.1	218.7	217.5	214.4	210.0	210.0
$E[(N_1 - q_1)^+]$	159.8	157.7	152.4	159.8	157.7	152.4	144.6	159.8	157.7	152.4	144.6	144.6
$E[(N_2 - q_2)^+]$	232.5	230.7	226.0	210.1	206.9	198.5	185.5	191.3	187.0	175.6	158.2	158.2
EP_1	34132.37	31063.35	24921.66	34132.37	31063.35	24921.66	18772.45	34132.37	31063.35	24921.66	18772.45	18772.45
EP_2	125612.35	113521.16	89333.94	125496.80	113403.43	89210.66	65004.81	125424.84	113329.85	89132.88	64920.58	64920.58
TEP	159744.72	144584.51	114255.60	159629.17	144466.78	114132.31	83777.27	159557.21	144393.20	114054.53	83693.03	83693.03

$k = 0.4$												
$\alpha_{21} = 0.1$				$\alpha_{21} = 0.5$				$\alpha_{21} = 0.7$				
$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0.7$
q_1	169.4	162.5	148.5	169.4	162.5	148.5	134.3	169.4	162.5	148.5	134.3	134.3
q_2	234.4	224.8	205.5	224.7	215.4	196.5	177.2	202.5	209.6	191.0	172.0	172.0
$E[(N_1 - q_1)^+]$	159.8	152.1	136.6	159.8	152.1	136.6	120.5	159.8	152.1	136.6	120.5	120.5
$E[(N_2 - q_2)^+]$	232.5	222.1	200.9	210.1	199.6	178.1	155.6	126.2	181.1	160.2	138.3	138.3
EP_1	34132.37	30989.50	24702.14	34132.37	30989.50	24702.14	18411.42	34132.37	30989.50	24702.14	18411.42	18411.42
EP_2	125612.35	113417.66	89026.00	125496.80	113304.78	88918.28	64526.46	125229.48	113234.65	88851.79	64463.68	64463.68
TEP	159744.72	144407.16	113728.13	159629.17	144294.28	113630.42	82937.87	159361.85	144224.15	113553.93	82875.10	82875.10

$k = 0.8$												
$\alpha_{21} = 0.1$				$\alpha_{21} = 0.5$				$\alpha_{21} = 0.7$				
$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0$	$\beta = 0.1$	$\beta = 0.3$	$\beta = 0.7$	$\beta = 0.7$
q_1	169.4	156.3	130.1	169.4	156.3	130.1	103.9	169.4	156.3	130.1	103.9	103.9
q_2	234.4	216.2	179.7	224.7	207.2	171.9	143.0	202.5	201.6	167.3	132.9	132.9
$E[(N_1 - q_1)^+]$	159.8	146.6	120.6	159.8	146.6	120.6	95.3	159.8	146.6	120.6	95.3	95.3
$E[(N_2 - q_2)^+]$	232.5	213.5	175.8	210.1	192.3	157.3	123.3	126.2	175.1	143.5	113.0	113.0
EP_1	34132.37	30915.51	24481.08	34132.37	30915.51	24481.08	18045.31	34132.37	30915.51	24481.08	18045.31	18045.31
EP_2	125612.35	113314.0	88716.2	125496.80	113205.9	88623.4	64039.6	125229.48	113139.1	88567.3	63994.8	63994.8
TEP	159744.7	144229.5	113197.3	159629.2	144121.4	113104.5	82084.9	159361.8	144054.6	113048.4	82040.1	82040.1

3.3. Summary of Findings

The focus of this chapter is on the single period inventory decisions in which customers have a right to return products in case of dissatisfaction. These returns are commercial returns so they are in an as-good-as new condition and can be resold directly after a testing and repackaging process. In case of a shortage of any of the variants, the customer may prefer not purchasing her preference or switch to a downward ranked variant. We show that the optimal order quantity of any variant depends on substitution probabilities, the substitutable variant's price and the net demand function.

With a computational study, we explore the effects of return and substitution, by using two variants having independent normal distribution. This example shows that optimal order quantities decrease as the expected probability of returns or the expected probability of returns, that can be resold, increases. Under each case, the main reason for the decrease in the optimal order quantities is that the mean net demand, which is the expected demand, that can be satisfied from products which are sold but not going to return decreases as the probability of returns or with the expected probability of resalable returns increase. On the other hand, the price of net demand rises as either the return probability or the resalable probability increases. The total profit also declines as a result of increasing either one of these probabilities.

In the following section, we will modify our current model to analyze the retailer operational decisions, such as order quantities and pricing, in an environment where consumers have a tendency to avoid any losses while making their decisions, i.e. loss-

averse customers.

Chapter 4

Pricing and Customer Returns

Policies with Loss Averse Customers

4.1. Introduction

In the previous chapter, we have studied a model in which the retailer sells his products to risk-neutral customers whose valuations are uncertain before the purchase. The assumption that customers are risk-neutral, while making their purchasing decisions, is common in literature of consumer behavior. However, there are many cases in which consumers focus more on possible losses than gains and show a tendency towards avoiding losses. In the existing literature, studies such as Kahneman & Tversky (1979), Thaler

(1980) and Kahneman et al. (1991) show that consumers are more sensitive to losses than gains so they strongly prefer avoiding losses to acquiring gains. This implies that a person who incurs a loss of \$100 loses more satisfaction than that gains via a \$100 windfall. This phenomenon which is called *loss-aversion* is empirically well-established under different settings and it is been observed that it substantially affects consumer behavior in making purchasing decisions. For example, Putler (1992) analyzes the effect of loss-aversion by studying the consumers' response to egg price changes and finds that the price-increase elasticity is about two and half times higher than the price-decrease elasticity. Furthermore, during the period that the credit-card companies charged transaction fees to each card purchase, Thaler (1980) noted that the credit-card companies insist that any price difference between cash and card purchases should be labeled a cash discount rather than a credit surcharge since the same price difference is perceived as a gain in the former case but as a loss in the latter. He showed that from a customer point of view, it is always easier to give up a discount than to accept a surcharge. This is the reason why "late registration fee" is advertised more often than "discount for early registrations" to encourage early registrations for events. In this chapter, we relax the assumption of risk-neutral consumers and assume that customers are loss-averse in their decision making.

The other assumption that we relax in this chapter is the type of the return policy. Previously, we assume that the retailer provides a full-refund to each customer who returns the product. As discussed in Chapter 1, there are different types of returns which may have both positive and negative effects on the retailer's revenue. While

all return policies may stimulate purchases by protecting customers against product misfits and increasing customer satisfactions, they may have a negative effect on the retailer's revenue by yielding increases in inventories and handling costs. In view of such tradeoffs, the goal of the retailer is to offer a return policy which attracts customers and reduces their risks, without resulting in excessive return related costs to the retailer. While designing a return policy, the retailer should pay close attention to customers' purchasing behavior. Thus, ignoring loss-aversion of customers may result in a poor return policy and may affect the retailer's profit negatively.

In this chapter, we examine the situation in which a retailer sells a single product to loss-averse consumers. Different from the previous model, where the price of the product is exogenously determined and the return type is fixed, in the current model, the retailer makes both price and quantity decisions and also designs an appropriate returns policy in order to maximize his profit. Return policies differ in terms of their refund amount. If the retailer prefers a full-refund policy, then he reimburses the consumer the full price of the product, thus the refund amount is equal to the price of the product. Whereas, if a partial-refund policy is offered, the retailer offers a refund which is strictly less than the purchasing price. The retailer may also choose not to accept any returns, i.e. a no-refund policy implying a refund amount of zero. If the retailer chooses to offer a partial-refund policy instead of full- or no-refund policy, he needs to find the optimal refund amount. The retailer's policy concerning returns and pricing directly influences consumers' purchase decisions. Each consumer who purchases at most one product faces an uncertainty in product valuation before she experiences the product. Like the retailer,

a consumer aims to maximize her utility of the product in question. The utility function of a consumer is a decreasing function of the price. Since consumers are loss-averse, they are more sensitive to losses than gains. We model consumer's loss-aversion with a utility function which is much steeper for losses than gains. The selection of the return policy affects a consumer's purchasing decisions via her expected utility. Under this setting, we explore the following research questions:

- What is the impact of loss aversion on the seller's price and order quantity decisions?
- How does loss aversion affect the equilibrium return policy?

The key contribution of this model is to study consumer return policies with loss-averse customers. The model will provide some insights about loss-averse customers' purchasing behavior and will show that ignoring loss averse consumers may have a negative effect on the seller's profit.

The rest of this paper is organized as follows: Section 4.2 introduces the basic model. In Section 4.3, we explore the equilibrium outcome when the seller chooses either a no-refund, a full-refund or a partial-refund policy with loss-averse consumers. Section 4.4 briefly summarizes the results of the proposed model.

4.2. Model Development

We consider a single-period classical newsvendor model in which a seller faces random market demand x with the distribution function $F(\cdot)$. Each consumer in the market purchases at most one product or may choose not to buy it. A consumer does not realize the true valuation of product until she purchases and experiences it. When a consumer makes purchasing decisions, only a probability distribution of valuations is known. We assume that the random valuations v are identically and independently distributed with the distribution function $G(\cdot)$. Both $F(\cdot)$ and $G(\cdot)$ are assumed to be continuous and differentiable. The valuation is not known before the customer experiences the product. After she experiences the product, the true valuation is realized and she may decide to return the product for a refund, i.e. $r > 0$.

The chronology of events is as follows. First, the retailer announces a selling price, p , order quantity, q , and refund amount, r . A constant production cost, c is incurred each unit produced. Second, the stochastic market demand, x , is realized and $\min(x, q)$ units are sold. Then, consumers who purchase the product observe their individual valuations and decide whether to keep it or to return it, if the retailer offers any return policy. We assume that returned products cannot be resold. Finally, all unsold and returned products are salvaged in a secondary market with a salvage value s . We assume $s < c$ in order to avoid trivial arbitrage opportunities.

Consumers are assumed to be loss-averse. As mentioned before, this characteristic of consumers' purchasing behavior results in a utility function that is steeper for losses

than for gains. A parameter $\lambda \geq 1$ is chosen to indicate the degree of loss aversion. A higher values of λ indicates consumers are more loss averse. If $\lambda = 1$, then consumers are risk neutral. We assume that consumers are homogeneous with the same degree of loss aversion. Since a consumer aims to maximize the expected utility ($U(.)$), she compares the utility of keeping the product, $U(v - p)$ with the utility of returning the product $U(r - p)$. Clearly, she would keep the product if and only if $v \geq r$. Consumers obtain the following utilities if they pay a price p and obtain a value v :

$$U(v, p) = \begin{cases} v - p, & \text{if } v \geq p; \\ \lambda(v - p), & \text{if } p > v \geq r; \\ \lambda(r - p), & \text{otherwise} \end{cases} \quad (4.1)$$

If the market demand is assumed to be a mass of infinitesimal consumers, the probability of consumers who will return the product is equal to $G(r)$. The retailer's profit function is, thus,

$$\begin{aligned} \Pi(p, q, r) &= [p\bar{G}(r) + (p - r + s)G(r)] E_{min}(x, q) + s(q - E_{min}(x, q)) - cq \\ &= [(p - s)\bar{G}(r) + (p - r)G(r)] E_{min}(x, q) - (c - s)q. \end{aligned} \quad (4.2)$$

This profit function consists of profit from sold products, salvage from excess inventory and returns, and the refund paid for returns and the production cost. In equation (4.2), each unit sold and kept by the consumer yields a profit p , whereas each returned item yields the amount $(p - r)$ from the consumer and s from salvaging it. Each unsold unit

also yields s . The last term is the retailer's production cost.

The existing literature on consumer returns generally assumes that consumers are risk neutral. The key distinguishing feature of our model is that we study the effect of loss averse consumers on the retailer's decisions when the latter offers a no-refund, full-refund or partial-refund policy. Recently, Su (2009) studies consumer returns by considering a full- and partial-refund policy for risk neutral consumers. In this study, we adopt Su (2009)'s model as a benchmark and demonstrate how consumer loss-aversion changes the seller's optimal price and quantity under the same return policies.

4.3. Consumer Returns for Loss Averse Customers

4.3.1 No-refund Policy

We begin our analysis with the case in which the seller does not offer any return policy to consumers when the product does not fit their needs. In other words, the refund r is set to zero. So, the seller's problem includes only determining the optimal price and order quantity.

When returns are not accepted, the customer buys the product if and only if her expected utility, expressed below, is positive:

$$EU^N(p; r = 0) = \int_0^p \lambda(v - p) dG(v) + \int_p^{+\infty} (v - p) dG(v) \quad (4.3)$$

$$= Ev - p + (\lambda - 1) \int_0^p (v - p) dG(v), \quad (4.4)$$

where Ev is the expected consumer valuation. Clearly, $Ev - p$ is the expected utility if consumers are risk neutral and the last (negative) term in the equation above indicates the impact of loss aversion. The larger the value of λ the greater is the utility loss. Thus, the retailer's profit is given by $\pi_N(p, q; r = 0) = (p - s)E \min(x, q) - (c - s)q$.

Theorem 4.3.1 *Under a no-refund policy, the optimal price p_λ^N is the unique solution of $EU^N(p; r = 0) = 0$, and the optimal order quantity $q_\lambda^N = \bar{F}^{-1} \left(\frac{c - s}{p_\lambda^N - s} \right)$.*

All proofs are provided in the appendix. If the retailer does not offer any refund. The consumers' expected utility is monotonically decreasing in p . Therefore, the retailer would charge the highest possible price and the corresponding quantity is the optimal newsvendor quantity. The following lemma demonstrates the impact of loss aversion on the optimal price and quantity decisions when a refund is not offered.

Lemma 4.3.2 *p_λ^N and q_λ^N both decrease with λ .*

Lemma 4.3.2 indicates that the seller charges a lower price and orders fewer products when facing more loss averse customers. Since loss averse customers pay more attention to losses, for a given price, the expected utility for such a customer is lower than that for a risk neutral one, which hinders the seller from charging higher prices. As a result of a lower selling price, the profit margin gets smaller and so does the understocking risk and the retailer would order a smaller quantity.

4.3.2 Full-refund Policy

Under the full-refund policy, the retailer offers a very generous full-price refund policy, i.e. $r = p$ in the case of a misfit. Thus, a consumer will never encounter negative utility. Her utility is $v - p$ if $v \geq p$, or zero otherwise. The expected utility is $EU^F(p; r = p) = \int_p^{+\infty} (v - p) dG(v)$. Since the consumer's utility is independent of the degree of loss aversion, the analysis becomes identical for both loss averse and risk neutral customers as in Su (2009), and the optimal decisions are independent of λ . The retailer's profit is given by $\pi^F(p, q; r = p) = (p - s)\bar{G}(p)E \min(X, q) - (c - s)q$. Under the full-refund policy, a customer have the opportunity to replace the product and obtain a reimbursement of full price and her expected utility is always greater than or equal to zero. Thus, her purchasing decision does not depend on the price. The retailer, consequently, chooses the price which maximizes his own profit.

Theorem 4.3.3 *Under full-refund policy, the optimal price p^F is the unique solution of $p^F = \operatorname{argmax}(p - s)\bar{G}(p)$, and the optimal quantity $q^F = \bar{F}^{-1} \left(\frac{c - s}{(p^F - s)\bar{G}(p^F)} \right)$.*

The proof is straightforward and is thus omitted. A major question a retailer faces is whether it is more profitable to allow returns with a full price refund or adopting an all-sales-final policy. Su (2009) shows that if consumers are risk neutral, the seller prefers full-refund to no-refund only when the salvage value is sufficiently high, or when the production cost c is sufficiently high. However, given the salvage value and production cost are the same, the consumers' degree of loss aversion also has an impact on the

comparison. In general, consumer loss aversion forces the seller to offer more generous return policies. With the following lemma, we show that there exists a threshold $\bar{\lambda}$, such that if consumers are more loss averse than $\bar{\lambda}$, then a full-refund policy yields a higher profit than a no-refund policy.

Lemma 4.3.4 *For any s and c , there exists a $\bar{\lambda}$, such that $\pi^F \geq \pi^N$ if $\lambda \geq \bar{\lambda}$.*

Lemma 4.3.4 suggests that when the market is populated by loss averse customers, then the retailer needs to offer an aggressive refund policy to compensate for the consumers' fear of risk. The threshold $\bar{\lambda}$ indicates how likely it is for the retailer to adopt a full refund policy in a loss averse market, which it may be affected by two variables: salvage value and production cost. Clearly, a retailer is more willing to offer a full-refund policy when he can obtain higher revenue from salvaging returned products, therefore $\bar{\lambda}$ is reduced when the product's salvage value gets higher, as shown in the lemma below. On the other hand, production cost does not affect $\bar{\lambda}$, since the prices under both full-refund and no-refund scenarios do not depend on production cost.

Lemma 4.3.5 *$\bar{\lambda}$ decreases to 1 as salvage value, s , increases, whereas it is not affected by the production cost, c .*

Next, we provide a numerical example to show the effects of salvage value and the production cost on the seller's optimal profit, under no-refund and full-refund policies. We assume that the consumer valuations are uniformly distributed between 0 and 1, i.e., $v \sim U(0, 1)$. The unit cost of the product is set to 0.2. The aggregate demand function

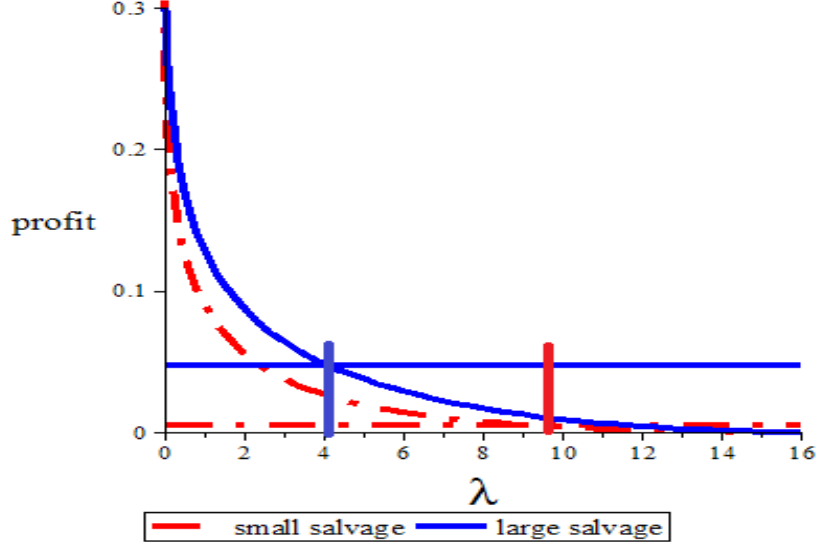


Figure 4.1: The seller's optimal profits with no-refund and partial-refund

is assumed to be a uniform distribution between 0 and 1, i.e., $x \sim U(0, 1)$. The uniform valuation function will allow us to have a closed form for the price and quantity variables. We will, then, be able to make a comparison. Su (2009) shows that when consumers are risk-neutral, the seller prefers the no-refund policy to the full-refund policy, when either the product cost is sufficiently low, or the salvage value is sufficiently low. Here, we will investigate numerically whether this result is valid when consumers are loss averse. Figure 4.1 shows the seller's optimal profits under both no-refund and full-refund policies for a small and large salvage value, i.e. $s = 0$ and $s = 0.15$, respectively and $c = 0.2$. As seen in this figure, when consumers are less loss averse, e.g. λ is small, the no-refund policy yields a higher profit than the full-refund policy. On the other hand, the full-refund policy dominates if consumers' loss aversion is high, e.g. λ is large. This result holds true for both small and large salvage values.

4.3.3 Partial-refund Policy

In this section, we will allow the seller to choose a refund between zero and the selling price. Such a partial-refund policy represents the case in which the seller imposes a restocking fee or a non-refundable charge on customers. Under this policy, the partial-refund amount r is incorporated as a new decision variable in the model.

The retailer maximizes his expected profit, given by equation (4.2), subject to the participation constraint $EU(p, r) \geq 0$, so that consumers are willing to buy the product. Given price p and refund r , consumers' expected utility from purchase can be represented as

$$EU(p, r) = \int_p^{+\infty} (v - p) dG(v) + \int_r^p \lambda(v - p) dG(v) + \int_0^r \lambda(r - p) dG(v) \quad (4.5)$$

$$= Ev - p + (\lambda - 1) \int_0^p (v - p) dG(v) + \lambda \int_0^r (r - v) dG(v). \quad (4.6)$$

Compared with the no-refund case, the last term above indicates the impact of a refund of r . This increases the expected utility by $E(r - v)^+$ in the case of a product misfit, adjusted by the degree of loss aversion, λ . Therefore, the refund provides customers some amount of relief, and the higher the refund, the more comfortable customers feel, while buying the product. With the partial-refund policy in effect, the problem for the retailer becomes more complex as he needs to manage two instruments - price and refund - in addition to the order quantity. The following theorem summarizes the optimal decisions:

Theorem 4.3.6 *Under a partial-refund policy, the seller's optimal price p_λ^P and optimal*

refund r_λ^P are the solutions to the following set of equations:

$$\begin{aligned}\lambda G(r) - [(r - s)g(r) + G(r)][1 + (\lambda - 1)G(p)] &= 0, \\ Ev - p + (\lambda - 1) \int_0^p (v - p)dG(v) + \lambda \int_0^r (r - v)dG(v) &= 0.\end{aligned}$$

and the optimal order is quantity $q_\lambda^P = \bar{F}^{-1} \left(\frac{c - s}{(p_\lambda^P - r_\lambda^P) G(r_\lambda^P) + (p_\lambda^P - s) \bar{G}(r_\lambda^P)} \right)$.

Note that in Su (2009), when customers are risk neutral, the optimal refund under a partial-refund policy is always the same as the salvage value, $r^* = s$. When consumers are loss averse, clearly the optimal refund amount is no longer fixed and will depend on λ . An interesting question is, then, whether the firm should refund more or less than the salvage value under this circumstance. Lemma 4.3.7 shows that the firm should offer a refund higher than salvage value when customers are loss averse.

Lemma 4.3.7 *The optimal refund $r_\lambda^P \geq s$.*

To gain more insight into how consumers loss aversion changes the optimal price and the order quantity, Lemma 4.3.8 below outlines analyzes how the loss-aversion degree affects these decision variables quantity:

Lemma 4.3.8 *Under a partial refund policy, (1) if we fix p , then the optimal refund r_λ^P and quantity q_λ^P increase in λ ; (2) if we fix r , then the optimal price p_λ^P and quantity q_λ^P decrease in λ .*

This Lemma 4.3.8 shows the impact of the degree of loss aversion if the retailer can only control one marketing instrument (either price or the refund amount). If price is fixed, then the retailer offers a more generous refund amount if customers are more concerned about the possibility of misfit. Similarly, for any refund amount r , the retailer charges a lower price and orders less quantity with a partial-refund policy, when customers are loss averse than when they are risk neutral. This result is not surprising since loss averse consumers are more concerned about their losses and their willingness to pay is less than that of risk neutral consumers. So, the retailer needs to charge a lower price to attract these consumers, if he is not willing to change the refund. If the retailer optimizes the refund along with the price and quantity, then his response to a change in customer behavior will not necessarily be the same as stated in Lemma 4.3.8. We will analyze the retailer's behavior for uniformly distributed customers valuations and uniform demand.

For the numerical example, we use the same setting as used in the previous section. Table 4.1 shows the optimal prices, quantities, profits and refund for partial-refund and full-refund policies. As Su (2009) indicates for risk-neutral consumers, the profit and the order quantity under the partial-refund policy are always higher than those with the full-refund policy, since the partial refund is strictly less than the full price. These results are valid for all consumers types with different degrees of loss-aversion degree. As seen in Table 4.1, the difference between the profits resulting from the partial- and full-refund policies diminishes as consumers' loss aversion increases. In addition, the optimal refund under the partial-refund policy converges to the optimal price as the degree of loss-aversion degree increases. In other words, as consumers become more

Table 4.1: Partial- and Full-refund Policies

$s = 0$										
	$\lambda = 3$		$\lambda = 10$		$\lambda = 50$		$\lambda = 100$		$\lambda = 1000$	
	Partial	Full	Partial	Full	Partial	Full	Partial	Full	Partial	Full
p	0.366	0.500	0.446	0.500	0.485	0.500	0.493	0.500	0.500	0.500
r	0.000	0.500	0.410	0.500	0.480	0.500	0.490	0.500	0.500	0.500
q	0.450	0.200	0.280	0.200	0.220	0.200	0.210	0.200	0.200	0.200
Π	0.038	0.005	0.011	0.005	0.006	0.005	0.005	0.005	0.005	0.005
$s = 0.08$										
	$\lambda = 3$		$\lambda = 10$		$\lambda = 50$		$\lambda = 100$		$\lambda = 1000$	
	Partial	Full	Partial	Full	Partial	Full	Partial	Full	Partial	Full
p	0.411	0.540	0.496	0.540	0.534	0.540	0.532	0.540	0.540	0.540
r	0.230	0.540	0.470	0.540	0.530	0.540	0.530	0.540	0.540	0.540
q	0.590	0.430	0.480	0.430	0.440	0.430	0.440	0.430	0.430	0.430
Π	0.052	0.020	0.027	0.020	0.021	0.020	0.020	0.020	0.020	0.020
$s = 0.15$										
	$\lambda = 3$		$\lambda = 10$		$\lambda = 50$		$\lambda = 100$		$\lambda = 1000$	
	Partial	Full	Partial	Full	Partial	Full	Partial	Full	Partial	Full
p	0.461	0.575	0.540	0.575	0.563	0.575	0.572	0.575	0.570	0.575
r	0.340	0.575	0.520	0.575	0.560	0.575	0.570	0.575	0.570	0.575
q	0.800	0.723	0.750	0.723	0.730	0.723	0.730	0.723	0.730	0.723
Π	0.078	0.047	0.055	0.047	0.049	0.047	0.048	0.047	0.047	0.047
$s = c = 0.20$										
	$\lambda = 3$		$\lambda = 10$		$\lambda = 50$		$\lambda = 100$		$\lambda = 1000$	
	Partial	Full	Partial	Full	Partial	Full	Partial	Full	Partial	Full
p	0.495	0.600	0.567	0.600	0.593	0.600	0.601	0.600	0.600	0.600
r	0.400	0.600	0.550	0.600	0.590	0.600	0.600	0.600	0.600	0.600
q	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Π	0.107	0.080	0.087	0.080	0.081	0.080	0.081	0.080	0.080	0.080

sensitive to their losses, the seller offers more generous refunds under a partial-refund policy. Meanwhile, an increase in the refund amount has a positive effect on the price. So, both the price and refund amount rise as consumers get more loss averse. The partial refund amount can increase to a full refund amount, equal to the price. Thus, for a comparatively large degree of loss-aversion, the partial-refund amount approaches the full price.

Another result that can be seen in Table 4.1 is the behavior of the optimal price and refund amount. In Lemma 4.3.8, we show that the optimal price and quantity decrease with an increase in loss-aversion if the seller is not willing to change his refund amount. This result is no longer valid if the seller optimizes the refund along with the price and quantity. As seen in this table, the optimal refund increases as consumers become more loss averse. In addition, the optimal price increases due to the increase in the refund. This behavior is analytically shown for a uniform valuation distribution in the Appendix. However, the revenue from returns does not increase in this case. In addition, either the optimal order quantity or the profit is not positively affected by the increase in price and refund.

If we compare the three return policies, we can find that both the optimal price and the refund amount follow the orders shown in the lemma below. The order with regard to optimal quantity depends on the relevant parameters.

Lemma 4.3.9 $p_\lambda^N \leq p_\lambda^P \leq p^F$, and $r^N \leq r_\lambda^P \leq r^F$.

This lemma shows that better return policies yield higher return refunds.

4.4. Summary of Findings

In this chapter, we analyze consumers returns policies when consumers are more sensitive to losses than gains. Our major findings are summarized below:

- Regardless of return policies, the seller needs to compensate consumers' sensitivity to losses by lowering the purchase price.
- Loss-aversion forces the seller to keep less inventory.
- As consumers get more loss-averse, the full-refund policy becomes more profitable to the seller if a partial-refund is not an option.
- Under the partial-refund policy, the seller needs to offer a larger refund if consumers are loss-averse, than when they are risk neutral.
- Under the partial-refund policy, the optimal refund converges to the optimal price as the degree of loss-aversion degree increases.
- The optimal refund and price may increase as consumers become more loss averse, while the optimal quantity and profit decrease.

In the next chapter, we will continue to analyze consumer returns by modifying the models presented in Chapter 3 and in this chapter. In Chapter 5, we extend those models to discussed the effects of a competitive market on the seller's economic decisions. We discuss the equilibrium policy, when two competing sellers use return policies to increase their market share. As like in the basic model, we assume the market consists of risk-neutral consumers who wish to optimize their utility from the product they are about to purchase.

Chapter 5

Pricing and Customer Returns

Policies for Two Competitive

Retailers

5.1. Introduction

In the previous chapters, we have analyzed how consumer returns have an impact on a monopolistic retailer's operational and pricing decisions under different scenarios. The majority of the research in consumer returns, like our suggested models, above has been confined to monopolistic scenarios. However, competition is often pervasive in many market environment. In such an environment, a customer's task of assessing the value of a product is more challenging, since the true value of the product is affected

by several additional variables a part from the product itself. For instance, although different firms sell the same product, their respective customer service levels, shipping or packaging options etc may differ and affect each customer's utility of a product offered by a retailer in a different way. In other words, a specific customer may prefer to have 24-hour customer service, while another may like to have a choice of shipping options. Thus, a customer-friendly return policy serves as a competitive advantage for a retailer. This chapter aims to study the equilibrium return policies in such a competitive market.

In this chapter, we will relax the assumption of a monopolistic market made in the previous models, and investigate how a retailer's pricing changes in the presence of returns. We examine a scenario where two competing sellers sell a homogeneous product to end customers at the retail level. Each seller independently determines the product's purchase price and return policy, in order to maximize his own profit. We first analyze the case where each seller offers either a no-refund (i.e. all sales are final), or a full-refund option to customers. We also allow sellers to choose a partial-refund policy with a refund that is strictly less than the purchase price. Needless to say that a retailer's policy about returns and product pricing directly influence the customers' purchase decisions, thus affecting overall product demand. Each customer who purchases at most one product from only one of the two sellers faces an uncertainty in product valuation, which generally occurs before she has had a chance to experience the product. As in the case of a retailer, a rational customer attempts to maximize her own expected utility of the product in question. When a customer decides to purchase the product, she tends to have some disutility in choosing one of the two firms in addition to the consideration of purchase

price. The disutility of a product results from the difference between the services offered by the retailers and their individual characteristics. As mentioned above, each retailer has unique characteristics such as service level, shipping options etc. and customers' preferences for those services may be different. This disutility of a product reflects the nature of competition between firms. If sellers are similar (or different) in terms of such characteristics, we can say that there is an intense (or moderate) competition between them. We assume that all customers and the two retailers are located along a straight line and the disutility is measured by the distance between a customer and the retail location from which the product may be purchased, in each instance. As a result, the market size of each seller depends on his own and the rival's price and return policies, customer valuations of the product and the degree of competition between the sellers. It is assumed that the sellers do not have capacity restrictions. In other words, each seller can cover the whole market by himself. Thus, pricing and return policy decisions are likely to play important roles in the division of the total market.

This chapter presents a model that investigates the effects of return policies on each of the two sellers' pricing decisions, when they engage in market size competition. Our model simultaneously addresses a consumer's purchase decision and the competitive firms' pricing decisions, along with their respective return policies. We explore the following major research questions:

- When the sellers are only allowed to offer a full refund or none at all, what is the equilibrium return policy? How does the equilibrium return policy depend on

other factors, such as salvage value and competition level?

- When the sellers are allowed to offer partial refunds, what is the equilibrium return policy?
- How does the return policy affect equilibrium prices?
- What return policy results in the highest consumer surplus?

Our key contribution of this model is to provide some insights into consumer return policies in a competitive environment. Our suggested model is suitable for different scenarios involving the market. For instance, the model can be used even the retailers sell substitute products to consumers with different valuations.

The rest of this chapter is organized as follows: Section 5.2 presents the detailed development of our model. In Section 5.3, we explore the equilibrium prices when each seller chooses to offer either a no-refund or a full-refund policy. Section 5.4 presents the results of a numerical study that compares the optimal prices and profits under different operating conditions, with no-refund and full-refund policies. In Section 5.5, we extend the model to the case of partial-refund policies where the refund amount is less than the purchase price. Section 5.6 summarizes the results found with the current model.

5.2. Model Development

We consider two sellers, A and B, that compete to sell a product to end consumers and we aim to study the equilibrium pricing and return decisions. We use a modified

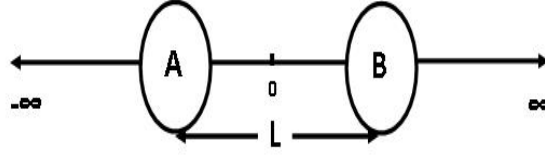


Figure 5.1: Competitive Market

version of the classical Hotelling's model which assumes that customers are uniformly distributed along a line with a population density of 1 and the two sellers are located at the two ends of this line. Note that one important benefit of a return policy is to increase sales. Under the Hotelling model, when prices are low enough such that the entire market is covered by the two sellers, total sales do not increase if both sellers offer return policies. Thus, the above mentioned benefit cannot be captured by such a model. Therefore, we modify Hotelling's model such that consumers are assumed to be uniformly distributed along a line $(-\infty, +\infty)$ and the sellers, A and B, are located at y_i , with $y_A = -\frac{L}{2}$ and $y_B = \frac{L}{2}$ where L is the distance between the sellers (see in Figure 5.1). Although we assume an infinite customer base, the "effective" market size is finite due to a bounded consumer valuation for the product and a travel cost incurred by each customer. Similar models are employed Shulman et al. (2009) and Pun & Heese (2010). As in the Hotelling model in Figure 5.1, L represents travel distance, or more generally, the degree of competition between the two sellers. A small L indicates that the two firms are quite similar with respect to service and other characteristics such as location and product variety. Thus, a relatively small L implies a case of intense competition between the sellers, while a larger L indicates that the competition is more moderate.

For analytical tractability, we restrict our attention to newsvendor-type products

with short life cycles, allowing us to use a single-period model. The relevant events occur in the following sequence. At the beginning of the selling season, the two sellers simultaneously determine their respective selling price, p_i , of the product and the refund amounts, r_i ($i = A, B$ respectively for the two sellers), in order to maximize their own expected profits. Customers have a random valuation v on the product and decide whether to buy at most one product and from which seller to purchase it. If a customer decides to buy a product from retailer i , she pays a price p_i . After she experiences the product, the true valuation is realized and she may decide to return the product and receive a refund of r_i . Each seller incurs a constant per unit production cost, c_i . Returned products are salvaged in a secondary market with a salvage value $s_i < p_i$ ($i = A, B$). The return inconvenience cost for customers and the handling cost for the retailers are omitted, although including them would not change the basic structure of the analytical results. We focus our attention on pricing and return policy decisions and do not incorporate the retailers' ordering decisions in this analysis.

As stated above, consumers are uniformly located along the line $(-\infty, \infty)$. They are homogeneous *ex ante* and do not know the true value of the product until they buy it and have the opportunity to use it. In this model, the product's value, v , is drawn from a strictly positive probability distribution having an increasing failure rate with cdf $G(\cdot)$ and pdf $g(\cdot)$, respectively, with a lower bound \underline{v} and an upper bound \bar{v} . In addition to the purchase price offered by a retailer, a consumer on the $(-\infty, +\infty)$ line incurs a travel cost (disutility) which is measured by the distance between her and the retailer. Without loss of generality, travel cost per unit of distance is assumed to be 1, thus a

consumer located at $x \in (-\infty, +\infty)$ incurs a travel cost of $|x + L/2|$, if she buys the product from seller A , and $|x - L/2|$ if she buys from seller B . Therefore, although we assume an infinite market line, only customers located between $(-L/2 - \bar{v}, L/2 + \bar{v})$ can possibly have positive utility from purchases, and thus, the effective market size is finite. Each consumer makes two sequential decisions. First, she decides whether to purchase the product and from which seller to purchase it. If she decides to buy it, she makes an additional decision whether or not to keep it if a return option is offered after the valuation is realized. If a customer is located at x , she can make one of the following three possible decisions:

- Buy from seller i and keep the product. The consumer realizes a value of v from the product, pays a price p_i and a transportation cost $|x - y_i|$, and thus, her net utility is $v - p_i - |x - y_i|$.
- Buy from seller i and subsequently return the product. The consumer pays price p_i incurs transportation cost $|x - y_i|$, and receives refund r_i , thus, her net utility is $r_i - p_i - |x - y_i|$.
- Do not buy the product, in which case the net utility is 0.

If a customer has purchased the product, she returns it if the net utility of returning it is higher than the net utility resulting from keeping the product. Therefore, if a customer located at x purchases from seller i , her expected net utility is $E\max(v, r_i) - p_i - |x - y_i|$. Customers then choose one of these three options to maximize expected utility: (1)

purchase from seller A and obtain an expected net utility $E\max(v, r_A) - p_A - |x + L/2|$;
 (2) purchase from seller B and obtain an expected net utility $E\max(v, r_B) - p_B - |x - L/2|$;
 and (3) do not purchase with a net utility of zero.

The two sellers make decisions in anticipation of customers' responses. First, we restrict our attention to two commonly observed return policies in practice: (1) all sales are final with no returns allowed and (2) a refund of the full purchase price is provided for a returned item. When the refund (r_A or r_B) is equal to zero (or the full purchase price), it is referred to as a *no-refund* (or a *full-refund*) policy. Later, in our analysis, we allow the refund amount to take any value between zero and the purchase price which is called a *partial-refund* policy. These three types of return policies are examined by Su (2009) in a monopolistic environment. Our work explores the characteristics of these policies in a competitive market, consisting of two retailers.

5.3. Equilibrium Prices under Given Return Policies

In this and the following sections, we consider two commonly used return policies: *no-refund* and *full-refund*. The possibility of a partial refund is explored in Section 5.5. We examine the equilibrium prices under both the return policies in this section, and analyze the equilibrium return policies in the next section. Hereafter, the letter N depicts a no-refund policy and F depicts a full-refund policy. In describing the refund policies simultaneously adopted by the two sellers, the first of a two-letter label indicates the refund policy adopted by seller A and the second letter indicates the policy adopted by

seller B. For example, NF represents the situation where seller A has a no-refund policy and seller B has a full-refund policy. Thus, there are four possible scenarios: NN, FF, NF, and FN, involving the refund policies of the two sellers.

In analyzing the equilibrium prices under different combinations of the sellers' refund policies, we find that the equilibrium prices depend on the distance, L , between the two sellers (i.e., the degree of competition). Under all four scenarios, when competition is intense, i.e., when L is small, there is a unique equilibrium price combination and the two sellers compete head to head. When competition is moderate, i.e., L is large, there is also a unique equilibrium price combination but each seller acts as a monopolist in its own market. On the other hand, if competition is at intermediate level, multiple price equilibria exist.

5.3.1 No-refund/No-refund (NN)

We begin our analysis with the case in which neither of the sellers offer refunds for returns. Each seller tries to maximize his profit function of $\Pi_i = (p_i - c_i)M_i$, $i = A, B$ where M_i is the market size of seller i . A customer located at x has three possible expected utilities for the product: $Ev - p_A - |x + L/2|$ if buying from seller A, $Ev - p_B - |x - L/2|$ if buying from seller B, and 0, if not buying from either seller. Thus, the market size of each seller depends on the relationship between the sellers' prices and the competition level, i.e., L , leading to three possible cases for market sizes:

Case 1 (seller A dominates): If $p_A \leq p_B - L$, then all consumers who are willing to

purchase the item prefer seller A. Thus, the market size of A is $M_A = 2(Ev - p_A)$ and the market size of B is $M_B = 0$.

Case 2: If $p_B - L < p_A \leq p_B + L$, then consumers' decisions depend on their expected utility values.

Case 2.1 (competition): If $Ev - p_A > \frac{1}{2}(L + p_B - p_A)$ (i.e., $p_A < 2Ev - p_B - L$), then consumers located on $(-\infty, -L/2)$ (or $(L/2, \infty)$) and consumers on $(0, L)$ prefer seller A (or B). The total market is shared by seller A and B, such that $M_A = Ev - p_A + \frac{1}{2}(L + p_B - p_A)$ and $M_B = Ev - p_B + \frac{1}{2}(L + p_A - p_B)$. Note that the market size of each seller depends not only on his own price but also on his competitor's price.

Case 2.2 (two monopolists): If $Ev - p_A < \frac{1}{2}(L + p_B - p_A)$ (i.e., $p_A > 2Ev - p_B - L$), then the market size of each seller is independent of the competitor's decision and $M_A = 2(Ev - p_A)$ and $M_B = 2(Ev - p_B)$.

Case 3 (seller B dominates): If $p_A > p_B + L$, all consumers willing to buy the product prefer seller B. Thus, $M_A = 0$ and $M_B = 2(Ev - p_B)$.

Figure 5.2 visualizes the relationship between the sellers' prices and their market sizes. Case 1 represents the situation where the price of seller B is significantly higher than that of seller A. So, seller A covers the whole market, while his rival has no market size. Case 3 represents the symmetric situation where seller B covers the entire market. In case 2, both sellers have positive market sizes. Case 2.1 occurs when the sellers'

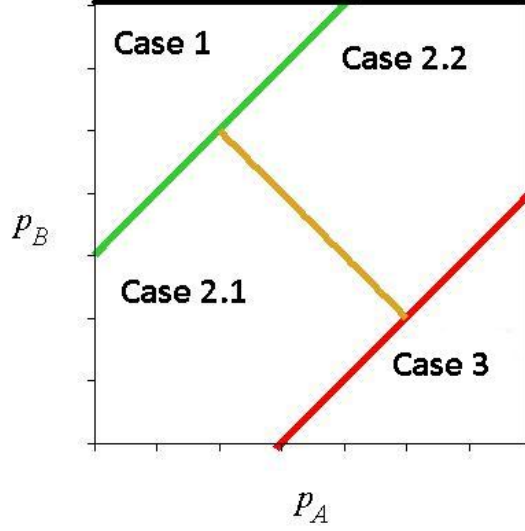


Figure 5.2: Cases under different price pairing

prices are similar and relatively low while Case 2.2 represents the scenario where both the firms' prices are similar, but relatively high. Thus, in Case 2.1 the whole market located between the two sellers is covered, and the two sellers compete to obtain market sizes on the basis their relative prices. In Case 2.2, the market located between the two sellers is not entirely covered, so each retailer acts as if he is a monopolist. Cases 1 and 3 are not of interest in this study, since we aim to study the interaction between two competing retailers. Moreover, we can argue that Case 1 and 3 cannot sustain the equilibrium prices if the unit costs c_A and c_B are close to each other.

Lemma 5.3.1 *As long as the unit costs c_A and c_B do not differ significantly, Case 1 and 3 cannot sustain the equilibrium prices.*

Base on this lemma 5.3.1, if seller A chooses a price larger than $p_B + L$, he will get a zero market size (i.e., zero profit). However, if he lowers his price, p_A , just below $p_B + L$,

he will capture a positive market size. Now, assume that p_B is the equilibrium price of seller B, then p_B should be larger than c_B . When the cost parameters are symmetric, $p_B + L - \epsilon > c_A$, then $p_A = p_B + L - \epsilon$ is a feasible solution for seller A. Therefore, $p_A > p_B + L$ is always dominated by $p_A = p_B + L - \epsilon$. Similarly, $p_B > p_A + L$ for seller B is always dominated by $p_B = p_A + L - \epsilon$. Thus, we restrict our attention to Case 2 only. Under Case 2, the profit function of seller A is:

$$\Pi_A(p_A, p_B) = \begin{cases} \pi_A^1 = (p_A - c_A)(Ev - p_A + \frac{1}{2}(L + p_B - p_A)), & \text{if } p_A \leq 2Ev - L - p_B; \\ \pi_A^2 = (p_A - c_A)2(Ev - p_A), & \text{otherwise.} \end{cases} \quad (5.1)$$

In equation (5.1), π_A^1 corresponds to Case 2.1 and π_A^2 corresponds to Case 2.2. Thus, the piecewise profit function results in a piecewise response function. In the following lemma, we summarize the response function of seller A given the price of firm B. Note that seller B has a symmetric response function.

Lemma 5.3.2 *Response Function of Seller A under NN Policies.*

Given p_B , the best response function $p_A^(p_B)$ is given by:*

$$p_A^*(p_B) = \begin{cases} \frac{1}{3}(Ev + \frac{1}{2}L + \frac{1}{2}p_B + \frac{3}{2}c_A), & \text{if } p_B \leq \frac{10Ev - 7L - 3c_A}{7}; \\ 2Ev - L - p_B, & \text{if } \frac{10Ev - 7L - 3c_A}{7} < p_B \leq \frac{6Ev - 4L - 2c_A}{4}; \\ \frac{1}{2}(Ev + c_A), & \text{if } \frac{6Ev - 4L - 2c_A}{4} < p_B. \end{cases} \quad (5.2)$$

All proofs are provided in the Appendix. Note that the response function is not monotonic in p_B . Lemma 5.3.3 characterizes the equilibrium prices for symmetric sellers, which depend on the degree of competition L .

Proposition 5.3.3 *Equilibrium Prices under NN Policies*

If all parameters are symmetric, then there exists a lower threshold value $\underline{L}^{NN} = \frac{6}{7}(Ev - c_A)$ and an upper threshold $\bar{L}^{NN} = Ev - c_A$ for the degree of competition L , such that the equilibrium prices are given by:

$$\left\{ \begin{array}{l} p_A^e = p_B^e = \frac{1}{5}(2Ev + L + 3c_A), \text{ if } L \leq \underline{L}^{NN}; \\ \text{any } p_A^e + p_B^e = 2Ev - L, \text{ such that } \frac{\partial \Pi_A^1}{\partial p_A}(2Ev - L - p_B^e) > 0, \text{ and } \frac{\partial \Pi_A^2}{\partial p_A}(2Ev - L - p_B^e) < 0, \\ \text{if } \underline{L}^{NN} < L \leq \bar{L}^{NN}; \\ p_A^e = p_B^e = \frac{1}{2}(Ev + c_A), \text{ if } L > \bar{L}^{NN}. \end{array} \right. \quad (5.3)$$

Lemma 5.3.3 states that there are three scenarios depending on the degree of competition L :

Scenario 1: Head-to-head Competition. When L is very small ($L \leq \underline{L}^{NN}$), the

two sellers are located very close to each other, or their distinctions in all other respects are small enough, so the two sellers engage in fierce price competition. The equilibrium prices increase in L , so the closer the two sellers are, the more intense the competition is and thus, the sellers need to lower their respective prices more. In this scenario, all customers located between the two sellers buy the product from one seller. Moreover, the customer located in the middle, $x = 0$, can obtain

strictly positive utilities from both sellers.

Scenario 2: Moderate Competition. When L has an intermediate value ($\underline{L}^{NN} < L \leq \bar{L}^{NN}$), the two sellers still engage in price competition. The whole market between the two sellers is also captured. The competition level, however, is moderate such that every customer can obtain strictly positive utility only from one of the sellers. Any combination of prices to achieve this may be the equilibrium prices. Thus multiple equilibria exist in this scenario. Interestingly, when the two sellers charge the same price, the equilibrium prices decrease in L .

Scenario 3: No Competition. When L is sufficiently large, the two sellers are located far away from each other, and they act as if they are monopolistic firms in their own markets. The equilibrium prices are the same as the monopolistic prices and are independent of L .

5.3.2 Full-refund/Full-refund (FF)

We now analyze the case in which each seller offers to refund the full purchase price of the product when it does not fit a customer's individual needs or tastes and is returned. A full-refund policy is basically a 100% money back guarantee. Such a policy is usually offered to signal high product quality, tending to ensure customer satisfaction.

Similar to the case in which both sellers offer no-refund policies, we restrict our attention to the case where both sellers have positive market sizes. Let $\tilde{p}_A(p_B)$ be the solution of $p_A = E \max(v, p_A) + E \max(v, p_B) - p_B - L$, which is the price of seller A, such

that a customer is indifferent in terms of buying from either seller. The profit function for seller A can be written as

$$\Pi_A(p_A, p_B) = \begin{cases} \Pi_A^1 = [p_A - c_A + (s_A - p_A)G(p_A)]\{E \max(v, p_A) - p_A \\ \quad + \frac{1}{2}[E \max(v, p_A) - E \max(v, p_B) + p_B - p_A + L]\}, \text{ if } p_A < \tilde{p}_A(p_B) \\ \Pi_A^2 = [p_A - c_A + (s_A - p_A)G(p_A)]2[E \max(v, p_A) - p_A], \text{ if } p_A > \tilde{p}_A(p_B). \end{cases} \quad (5.4)$$

In (5.4), $(s_A - p_A)$ is the unit cost of return and $(s_A - p_A)G(p_A)$ indicates the expected return cost to seller A. Note that, $\Pi_A(p_A, p_B)$ is continuous in \tilde{p}_A .

Before finding the equilibrium function, we first need to show that the profit function of each retailer is unimodal so that there exists a unique response function of price depending on the competitor's price.

Lemma 5.3.4 *Unimodality of the Profit Function under FF Policies*

Let $\bar{p}_A = \arg \max_{p_A} (p_A - s_A)\bar{G}(p_A)$ where $\bar{G}(p_A)$ represents the complementary cumulative distribution function, then the optimal $p_A^*(p_B) \leq \bar{p}_A$ and $\Pi_A(p_A, p_B)$ is unimodal in p_A for $p_A \leq \bar{p}_A$. Thus, there exists a unique $p_A^*(p_B)$.

The only condition that we need for unimodality of the profit functions is that the valuation distribution is an increasing failure rate (IFR) distribution, which is satisfied by many common probability distributions. The best response price is always lower than the optimal monopolistic price \bar{p}_A . Given this property, the response functions are continuous and unique as shown in the following lemma.

Lemma 5.3.5 *Response Function of Seller A under FF Policies*

There exists $\underline{p}_B \leq \bar{p}_B$ such that

$$p_A^*(p_B) = \begin{cases} \arg \max \Pi_A^1(p_A, p_B), & \text{if } p_B < \underline{p}_B; \\ \tilde{p}_A(p_B), & \text{if } \underline{p}_B < p_B < \bar{p}_B; \\ \arg \max \Pi_A^2(p_A, p_B), & \text{otherwise.} \end{cases} \quad (5.5)$$

Next, we examine the equilibrium prices. The structure of equilibrium prices is very similar to the case of both sellers offering no-refunds (NN). Depending on the level of competition intensity, there may exist multiple equilibria. When L is very large or very small, there exist unique equilibrium prices. Otherwise, multiple equilibria may exist. Denoting $p_A^1(p_B) = \arg \max \Pi_A^1(p_A, p_B)$ and $p_A^2(p_B) = \arg \max \Pi_A^2(p_A, p_B)$, the equilibrium prices are:

$$(p_A^{1e}, p_B^{1e}) \text{ solves } (p_A = p_A^1, p_B = p_B^1), \quad (5.6)$$

$$(p_A^{2e}, p_B^{2e}) \text{ solves } (p_A = p_A^2, p_B = p_B^2), \quad (5.7)$$

$$(p_A^e, p_B^e) \text{ solves } (p_A = \tilde{p}_A(p_B), p_B = \tilde{p}_B(p_A)). \quad (5.8)$$

Proposition 5.3.6 *Equilibrium Prices under FF Policies*

If all parameters are symmetric, then there exists $\underline{L}^{FF} \leq \bar{L}^{FF}$ such that the equilibrium

prices are:

$$(p_A^e, p_B^e) = \begin{cases} (p_A^{1e}, p_B^{1e}), & \text{if } L < \underline{L}^{FF}; \\ \text{any } p_A^e + p_B^e = E \max(v, p_A^e) + E \max(v, p_B^e) - L, \text{ such that } \frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) > 0, \\ \text{and } \frac{\partial \Pi_A^2}{\partial p_A}(\tilde{p}_A) < 0, & \text{if } \underline{L}^{FF} < L < \bar{L}^{FF}; \\ (p_A^{2e}, p_B^{2e}), & \text{if } \bar{L}^{FF} < L. \end{cases} \quad (5.9)$$

5.3.3 No-refund/Full-refund (NF)

When one seller does not offer any refund while the other offers a full refund, we can characterize the equilibrium prices similar to those in the NN and FF scenarios. There also exist two threshold values, \underline{L}^{NF} and \bar{L}^{NF} , such that three types of equilibrium prices may exist depending on the degree of competition L .

Proposition 5.3.7 *Equilibrium Prices for NF Policy*

There exists $\underline{L}^{NF} < \bar{L}^{NF}$ such that

$$(p_A^e, p_B^e) = \begin{cases} (\operatorname{argmax} \Pi_A^1, \operatorname{argmax} \Pi_B^1) & \text{if } L < \underline{L}^{NF}, \\ \text{any } p_A^e + p_B^e = E \max(v, p_B^e) + Ev - L, \text{ such that} \\ \frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) > 0, \text{ and } \frac{\partial \Pi_A^2}{\partial p_A}(\tilde{p}_A) < 0 & \text{if } \underline{L}^{NF} < L < \bar{L}^{NF}, \\ (\operatorname{argmax} \Pi_A^2, \operatorname{argmax} \Pi_B^2) & \text{if } \bar{L}^{NF} < L \end{cases}$$

where $\tilde{p}_A = Ev + E \max(v, p_B) - p_B - L$.

5.4. Equilibrium Return Decisions

We now examine the equilibrium return decisions. Since the equilibrium prices obtained in Section 5.3 do not have closed-form solutions, numerical methods are resorted to in this section. We assume that the consumer valuations are uniformly distributed between 0 and 1, i.e., $v \sim U(0, 1)$. The salvage values and unit costs of the product for both retailers are assumed to be equal, i.e., $s_A = s_B$ and $c_A = c_B$. We set the unit costs to 0.1 and change salvage values between 0 and the optimal value of price. Note that a salvage value greater than the purchase price is not meaningful. Finally, we assume L is between 0.1 and 0.9. We observe that for $L < 0.5$ (or, $L > 0.5$), there is intense (or moderate) competition between the sellers. Thus, we set $L = 0.3$ to represent a case of intense competition and to $L = 0.7$ depicting a case of moderate competition.

By examining the equilibrium prices obtained in Section 5.3, we address the following issues:

- Effect of a seller's, e.g. seller A's, own return policy on his price and profit.
- Effect of the rival's, e.g. seller B's, return policy on seller A's price and profit.
- Effect of the competition level on seller A's price.

We first analyze the effect of seller A's return policy on his equilibrium price and profit. Figure 5.3 shows the equilibrium prices of seller A (y -axis) for different salvage values (x -axis) when the competition is intense, i.e, $L = 0.3$.¹ From Figure 5.3, it is

¹Since the unit costs and the salvage values are equal for sellers, Figure 5.3 (Figure 5.4) also shows

clear that seller A's price is always higher under the full-refund policy compared to the no-refund policy, independent of his rival's return policy. This result is intuitive. Since a full-refund policy reduces a customer's risk, it increases her expected utility of purchasing the product. In other words, the consumer's reservation price for the product is high, which allows the retailer to set a higher price. Another result that can be seen in this figure is that the optimal prices of both retailers increase with the salvage value, except for the case in which both sellers offer no-refunds (NN), where the salvage value does not have any impact on the prices. On the other hand, when the retailers' characteristics differ significantly, i.e., L is large, as outlined in Section 5.3, each seller acts as a monopoly. Thus, a seller's optimal price is not affected by the rival seller's refund policy, i.e., the NN and NF (or alternately, FF and FN) policy combinations yield the same equilibrium prices. The figures for this situation are omitted for avoiding duplication.

Before analyzing the equilibrium profits of seller A, given that seller B offers no- and full-refund policies respectively, we need to examine two variables: market size and unit profit. Market size of a retailer increases as his return policy improves for a constant salvage value. Thus, when a seller offers full-refunds, he has a higher market size than in the case when he offers no-refund. On the other hand, a better return policy increases a consumer's propensity to return the product, i.e. increases the return probability. As a result, the per unit expected profit of the seller, i.e. $p_A - c_A + (s_A - r_A)G(r_A)$, is

the equilibrium prices (profit) of seller B. For example, the line NF represents the price of seller B and seller A offers a full-refund, seller B offers a no-refund policy.

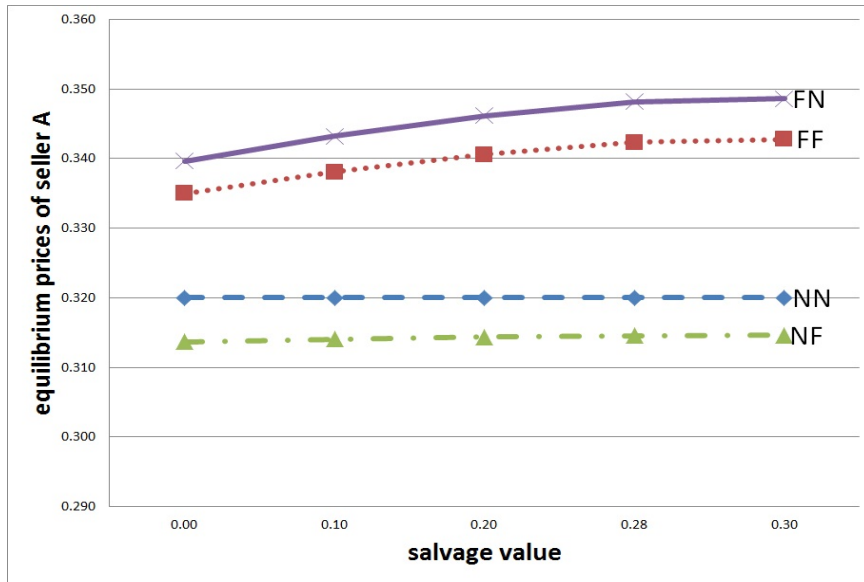


Figure 5.3: Equilibrium prices, $L = 0.3$, $c = 0.1$

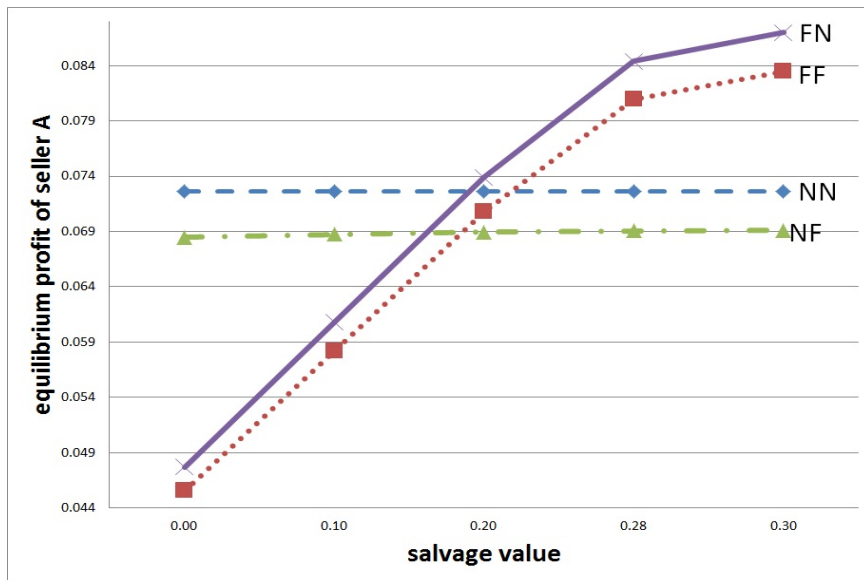


Figure 5.4: Equilibrium profits, $L = 0.3$, $c = 0.1$

negatively affected by an increase in the refund amount. Thus, the characteristics of the equilibrium profits depend on the relationship between the positive and negative effects of the refund amount. Figure 5.4 shows the equilibrium profits of seller A for $L = 0.3$. As seen in this figure, given the rival's return policy, the positive effect of seller A's full-refund policy outweighs its negative effect when the salvage value is high. In other words, when the salvage value is too small, a no-refund policy provides a higher profit for firm A; whereas when the salvage value is relatively large, a full-refund policy is preferable for seller A. When the salvage value is between its upper and lower limits, the equilibrium policy depends on the value of salvage and the competition level.

We now analyze the equilibrium policies in detail as a function of salvage value and L . Interestingly, we find that there may be a scenario where the phenomenon of Prisoner's Dilemma (Axelrod 1984) may occur. We first define the notation utilizes for describing the profit differences under different return policy combinations:

- Let $D_1(s, L)$ be the difference between the equilibrium profits of the NN and the FN policy combinations, i.e. $D_1(s, L) = \Pi_A^{e,NN} - \Pi_A^{e,FN}$, where $\Pi^{e,*}$ represents the equilibrium profit of '*' policy.
- Let $D_2(s, L)$ be the difference between the equilibrium profits of the FF and the NF policy combinations, i.e. $D_2(s, L) = \Pi_A^{e,FF} - \Pi_A^{e,NF}$.
- Let $D_3(s, L)$ be the difference between the equilibrium profits of the NN and the FF policy combinations, i.e. $D_3(s, L) = \Pi_A^{e,NN} - \Pi_A^{e,FF}$.

Thus, s_j is the unique solution of $D_j(s, L) = 0$, $j = 1, 2, 3$. It can be observed from Figure 5.4 that $s_3 > s_1 > s_2$. Thus, the equilibrium decision is to offer no-refunds (or alternately full-refund) for relatively low (or high) salvage values. However, there is an interval depending on the salvage value and L , such that although offering full-refund policies is an equilibrium for the sellers, they are better off if they offer no-refund policies. In other words, offering a full-refund policy is suboptimal for sellers. Thus, the prisoner's dilemma occurs in the corresponding interval (i.e. $s_1 < s \leq s_3$). Finally, symmetric policies in which both adopt either no-refund or full-refund policies are equilibrium policies for intermediate salvage values. These equilibrium decisions can be summarized as follows:

- Both sellers adopt no-refund policies when $s \leq s_2$.
- Both sellers adopt either no-refunds or full-refund model when $s_2 < s \leq s_1$.
- Both sellers adopt full-refund policies when $s_1 < s \leq s_3$ (Prisoner's Dilemma).
- Both sellers adopt full-refund policies when $s_3 < s$.

It is clear that the optimal equilibrium return policy depends on the salvage value. In practice, retailers have different ways of dealing with returned products. For example, returned food is often disposed of due to health and safety concerns, and software CDs are also disposed of, if the package is open, resulting in zero salvage value. Products such as clothes and electronics are sometimes sold with price markdowns, resulting in a moderate salvage value. Returned products that are in mint, as-new condition, such as

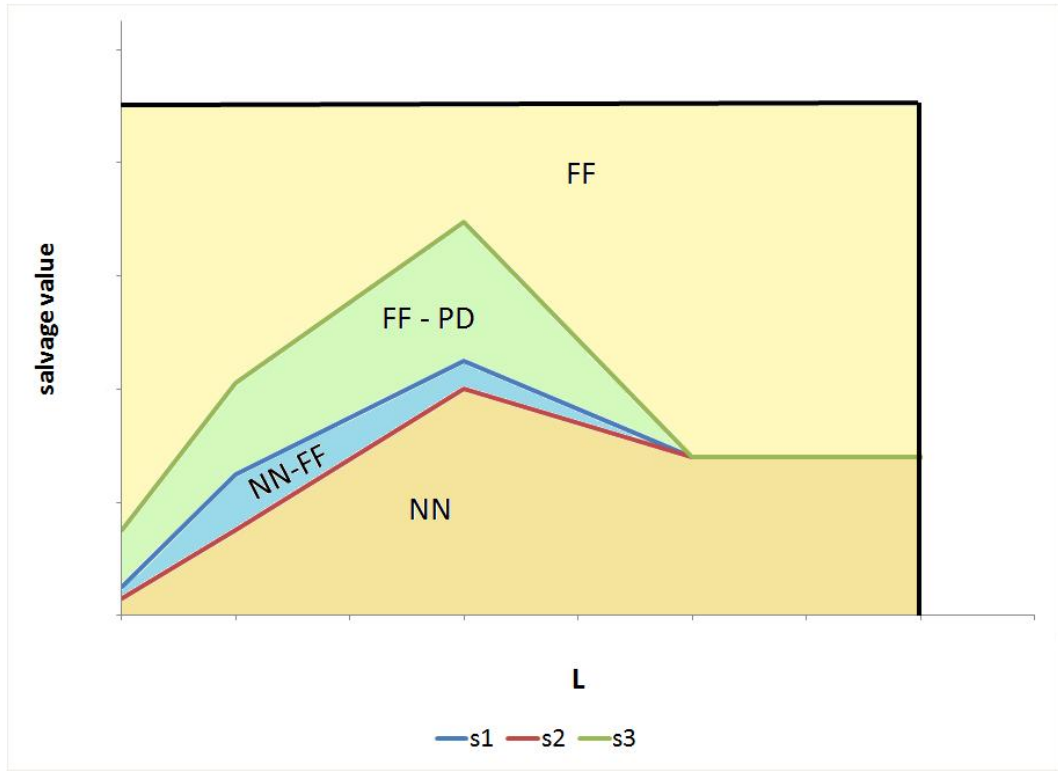


Figure 5.5: Equilibrium decisions (NN (FF): both sellers offer no(full)-refund; PD: Prisoner's Dilemma)

unworn shoes or unopened DVDs, they are often put back on the shelf to be sold to future customers at full price. The salvage values in such cases are the highest. Depending on different salvage values, the equilibrium refund policies should also be different. Many of the industry practices also confirm our results. For example, software products, once purchased, are often not allowed to be returned by consumers.

It is also observed that competition encourages the retailers to offer generous return policies (see in Figure 5.5). When L is large such that the two sellers act as monopolists, the threshold salvage value is lower compared to the case when L is small. Specifically, the threshold salvage is 0.173 for $L = 0.7$ vs 0.215 for $L = 0.3$.

Our final observation is the effect of the competition level on seller A's price. Let $\Delta_1 = (p_{FN} - p_{NN})/p_{NN}$ (alternately $\Delta_2 = (p_{NN} - p_{NF})/p_{NN}$) be the change in seller A's price if he (or his rival) offers a full-refund policy, and $\Delta_3 = (p_{FF} - p_{NN})/p_{NN}$ be the change in seller A's price if both sellers decide to offer full-refund policies. Figure 5.6 and 5.7 present these price changes under varying degrees of competition (i.e., L) for salvage values of 0.1 and 0.3, respectively. It has been noted before that as the competition becomes less intense, the market exhibits more monopolistic characteristics for each retailer. In this situation, if the salvage value is low, then the retailers do not have much motivation to improve their return policies. Instead, each seller prefers to use the price to attract customers which results in a price decrease. This can be seen in Figure 5.6 for Δ_1 and Δ_3 as the competition decreases from 0.1 to 0.3. When a threshold point for a competition level is reached, then retailers act as if they are monopolistic sellers in their own markets. The equilibrium prices are independent of L and the rival's strategy. When the salvage value is high, the seller tends to improve his return policy in order to benefit from salvaging returned items. Also, Figure 5.7 shows that Δ_1 and Δ_3 increases with the level of competition, i.e., if the competitor offers a better return policy, then the change in the seller's price increases.

5.5. Consumer Return Policies with Partial-Refund

We now extend our analysis to the case of partial-refund policies where the refund amount is strictly less than the retail price. A partial-refund occurs when a retailer

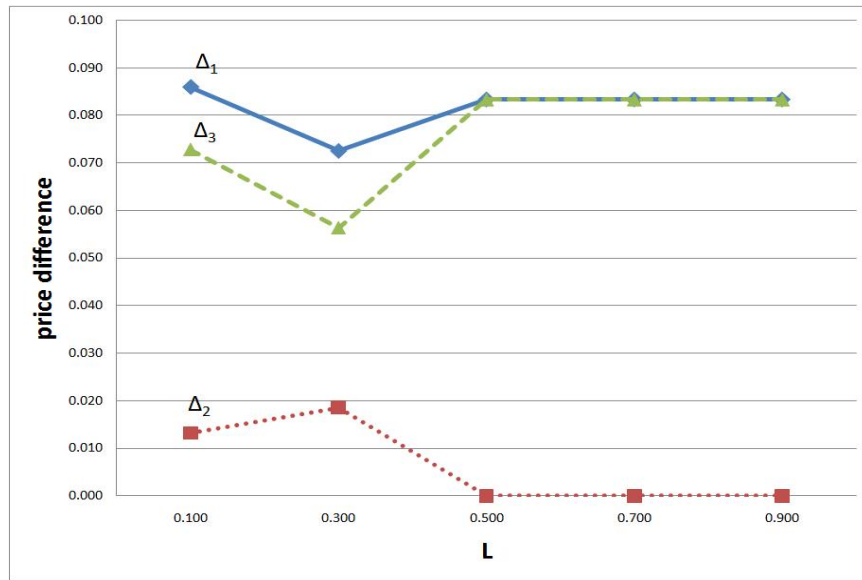


Figure 5.6: Price differences, $s = 0.1$

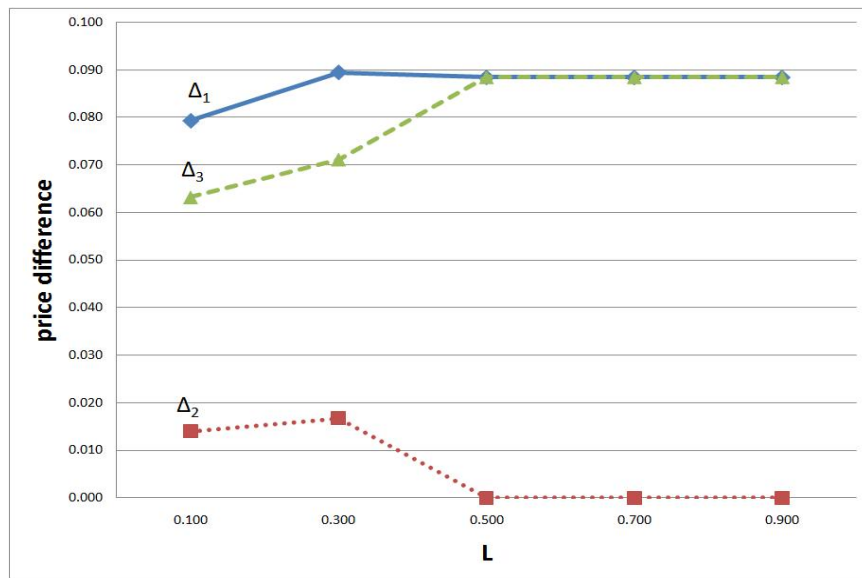


Figure 5.7: Price differences, $s = 0.3$

charges a restocking or a non-refundable fee, such as shipping or handling cost. The partial-refund amount is incorporated as a new decision variable in our model. As discussed in Section 5.4, a return refund has both positive and negative effects on a seller's profit. First, the product's expected net utility on the part of a consumer increases with the refund amount, allowing a seller to charge a higher price. On the other hand, a higher refund induces a higher return rate which negatively affects the profit of the retailer. Thus, when the return policy is either a no-refund or a full-refund policy, the retailer needs to find the optimal value of refund which considers a trade-off between its positive and negative effects. With our model, we investigate the effect of partial-refund policies on a seller's decision in a competitive market and compare it with no-refund and full-refund policies. Hereafter, the abbreviation P is used for depicting a partial-refund policy.

5.5.1 Partial-Refund/Partial-Refund (PP)

In this section, we analyze the case of both retailers offering partial-refunds. As in Section 5.4, we focus on the situation where both retailers earn positive market sizes.

Thus, the profit function of seller A is given by:

$$\Pi_A(p_A, p_B) = \begin{cases} \Pi_A^1 = [p_A - c_A + (s_A - r_A)G(r_A)]\{E \max(v, r_A) - p_A \\ \quad + \frac{1}{2}[E \max(v, r_A) - E \max(v, r_B) + p_B - p_A + L]\}, \text{ if } p_A < \tilde{p}_A(p_B, r_B, r_A), \\ \Pi_A^2 = [p_A - c_A + (s_A - r_A)G(r_A)]2[E \max(v, r_A) - p_A], \text{ if } p_A > \tilde{p}_A(p_B, r_B, r_A). \end{cases} \quad (5.10)$$

where $\tilde{p}_A(p_B, r_B, r_A) = E \max(v, r_A) + E \max(v, r_B) - p_B - L$ and $(s_A - r_A)G(r_A)$ is the expected return cost.

We first find the optimal refund amount for given retail prices. The following lemma indicates the optimal refund amount offered by a seller for a given price.

Proposition 5.5.1 *Optimal Refund Amount*

$r_A^* = s_A$ and $r_B^* = s_B$ for any $p_A \geq c_A$ and $p_B \geq c_B$, respectively.

Su (2009) shows that the partial refund is equal to the salvage value for a monopolistic seller. Our analysis shows that this result can be extended to a competitive market, and that it holds independent of a retailer's own, as well as his competitor's price.

Now, we derive the equilibrium prices depending on the level competition. The following lemma presents the equilibrium prices when the cost parameters are symmetric.

Proposition 5.5.2 *Equilibrium Prices under PP Policies* In the symmetric case,

there exists $\underline{L}^{PP} = \frac{6}{7}(E \max(v, s) - c)$ and $\bar{L}^{PP} = E \max(v, s) - c$ such that

$$(p_A^e, p_B^e) = \begin{cases} \frac{1}{5}(2E \max(v, s_A) + L + 3c_A), & \text{if } L < \underline{L}^{PP}; \\ \text{any } p_A^e + p_B^e = 2E \max(v, s_A) - L, & \text{such that} \\ \frac{1}{2}(E \max(v, s_A) + c_A) < p_A^e < \frac{1}{7}(4E \max(v, s_A) + 3c_A), & \text{if } \underline{L}^{PP} < L < \bar{L}^{PP}; \\ \frac{1}{2}(E \max(v, s_A) + c_A), & \text{if } \bar{L}^{PP} < L. \end{cases} \quad (5.11)$$

As discussed in Section 5.4, there are three possible scenarios depending on the level of competition. When L is small and competition is intense, both retailers compete

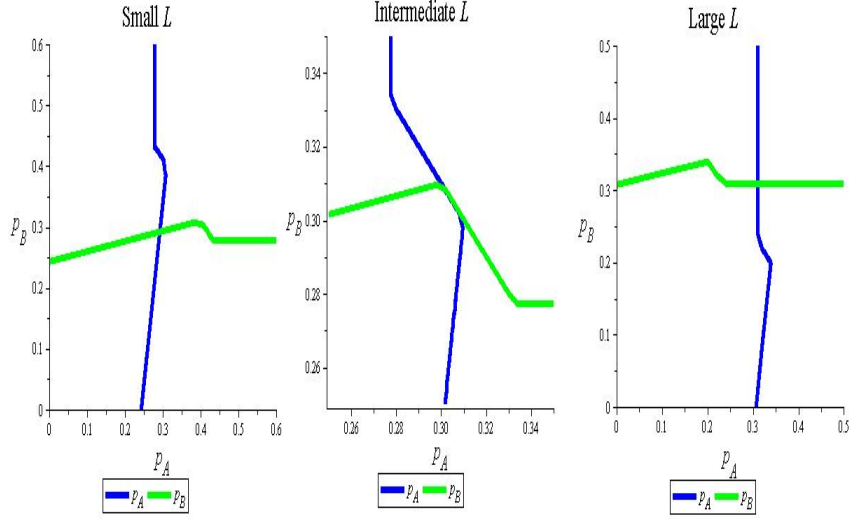


Figure 5.8: Equilibrium prices when both sellers offer partial-refund

head-to-head, and when L is large, each acts as a monopolist. For intermediate L , there may exist multiple equilibria. Figure 5.8 shows behavior of prices when both retailers offer partial-refund under different degrees of competition. This example is constructed when customers valuations are uniformly distributed between 0 and 1.

The equilibrium prices for asymmetric return policies including a partial-refund policy, i.e., PF and PN, are analyzed in the Appendix.

5.5.2 Numerical Study with Partial-Refund

In this section, we consider the combination of three return policies and explore the following issues:

- Effects of a seller's, i.e. seller A's, own return policy on his price and profit.

- Effects of seller A's competitor's, i.e. seller B's, return policy on his price and profit.
- The changes in consumer surplus and social welfare.
- The effects of production cost.
- Seller A's decision when a full-refund is offered or no-refund is not an option.

As in Section 5.4, the consumer valuations are uniformly distributed between 0 and 1. Also, the salvage values and costs are symmetric ($c = 0.1$) for the two sellers. Since the case of intense competition is more interesting, we set $L = 0.3$.

Effect of seller A's return policy on his price and profit:

Figure 5.9 (or alternately Figure 5.10) shows the equilibrium prices of seller A versus salvage values, given seller B's adoption of a no-refund (or partial-refund) policy when the competition is intense, i.e., $L = 0.3$. The graphs for the case when seller B offers a full-refund policy under moderate competition are not included here, since both of these situations have exactly the same patterns as in the case when seller B offers no-refunds. As seen in these figures, a higher refund amount results in a higher price. Thus, the price is always higher with full-refund than under the partial- and no-refund policies. Similarly, a partial-refund policy always leads to a price which is higher than that under a no-refund policy. The effect of the salvage value is positive as described in Section 5.4, except for the case in which seller A does not offer any refund, while seller B offers a partial-refund policy (i.e., the NP policy combination). In this case, seller B offers a

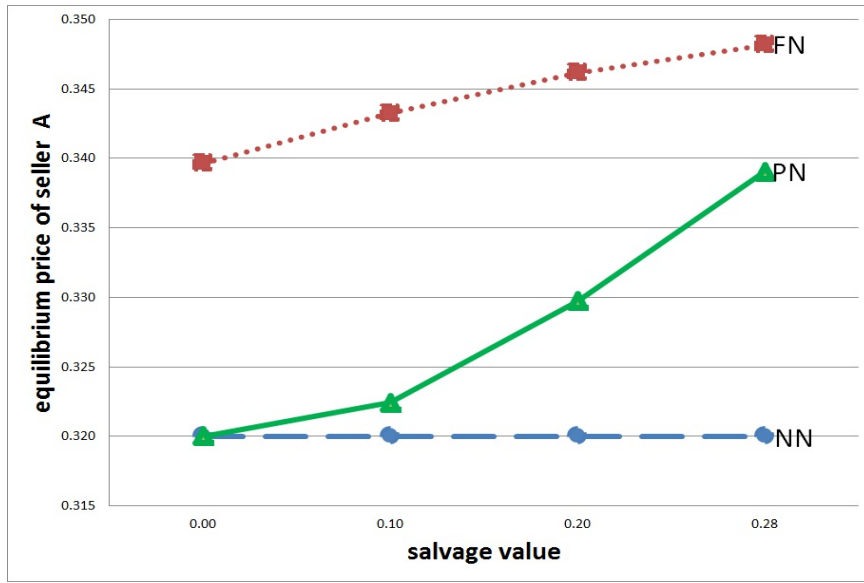


Figure 5.9: seller A's price given seller B offers no-refund, $L = 0.3, c = 0.1$

larger refund amount when the product can be salvaged at a higher value. Therefore seller A needs to lower his price to attract consumers, since he cannot do so with a no-refund policy.

Figure 5.11 (or alternately Figure 5.12) shows the equilibrium profits of seller A, given that seller B offers no-refund (or a partial-refund) for $L = 0.3$. As seen in these figures, the positive effect of a high refund amount with partial-refunds always outweighs its negative effect. Although the partial-refund policy is not as attractive to the consumer as the full-refund policy (the optimal partial-refund amount is less than the full-refund), the negative effect of full-refund on unit profit dominates its positive effect, in terms of market size. A partial-refund policy generates a larger market size and a higher unit profit in comparison with a no-refund policy. Therefore, the equilibrium refund policy is always a PP policy combination, i.e., both sellers offer partial refund policies. In addition, Figures 5.11 and 5.12 show that as salvage value increases, seller A's profit

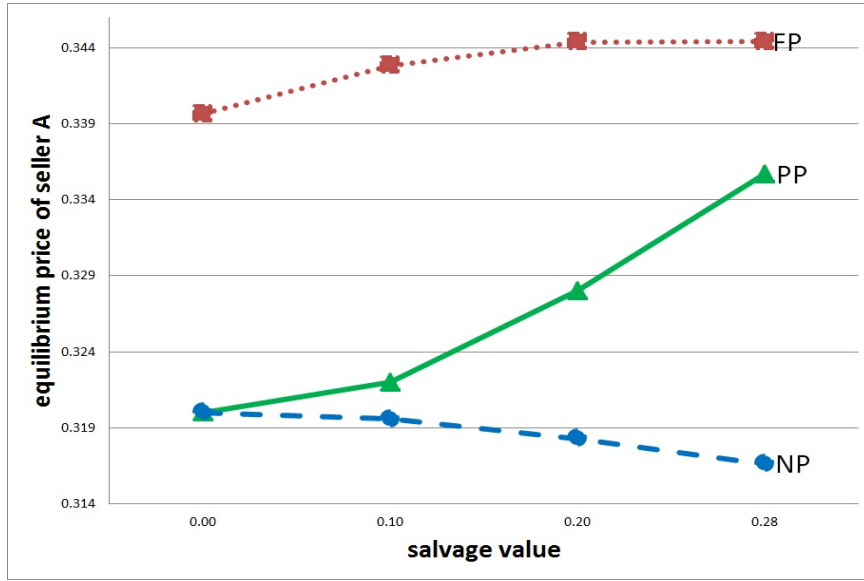


Figure 5.10: seller A's price given seller B offers partial-refund, $L = 0.3, c = 0.1$

increases, except for the case when he offers a no-refund policy, while his rival offers a partial-refund (i.e., the NP policy combination). In this case, when seller A decreases his price, his profit decreases as the salvage value increases. Note that both the price and profit decrease with the salvage value in the NP case, where the assumption is that both sellers have the same salvage value.

Effect of seller B's return policy on seller A's price and profit:

Next, we analyze the effect of seller B's return policy on seller A's price and profit. Figures 5.13, 5.15 and 5.17 (or alternately, Figures 5.14, 5.16 and 5.18) show the impact of seller B's policy on equilibrium price (or profit) of seller A when seller A offers no-, full- and partial-refund policies, respectively. These figures show that the effect of seller B's policy on seller A's equilibrium price and profit vary with seller A's decisions. In Figures 5.13 and 5.14, we can see that when seller A decides not to offer any refunds, then he sets the highest price and has the highest profit when both retailers choose the

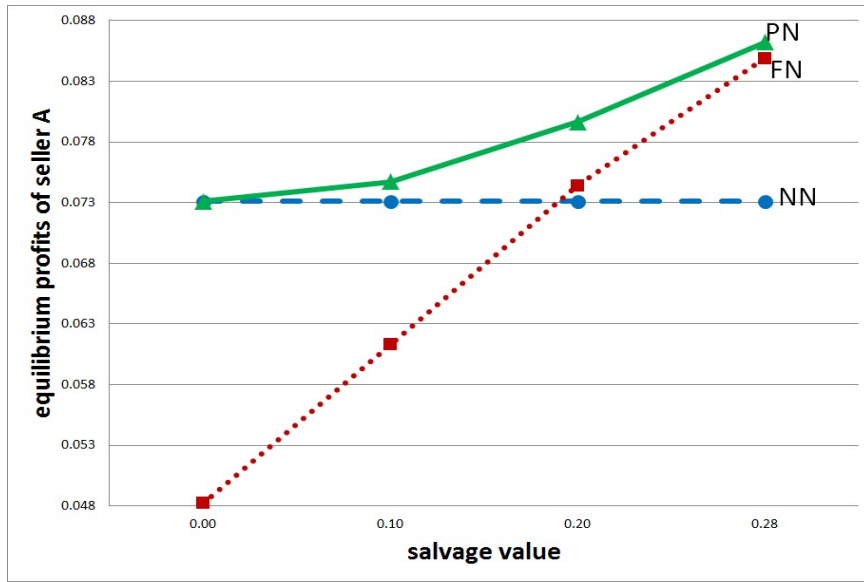


Figure 5.11: seller A's profits when seller B offers no-refund, $L = 0.3, c = 0.1$

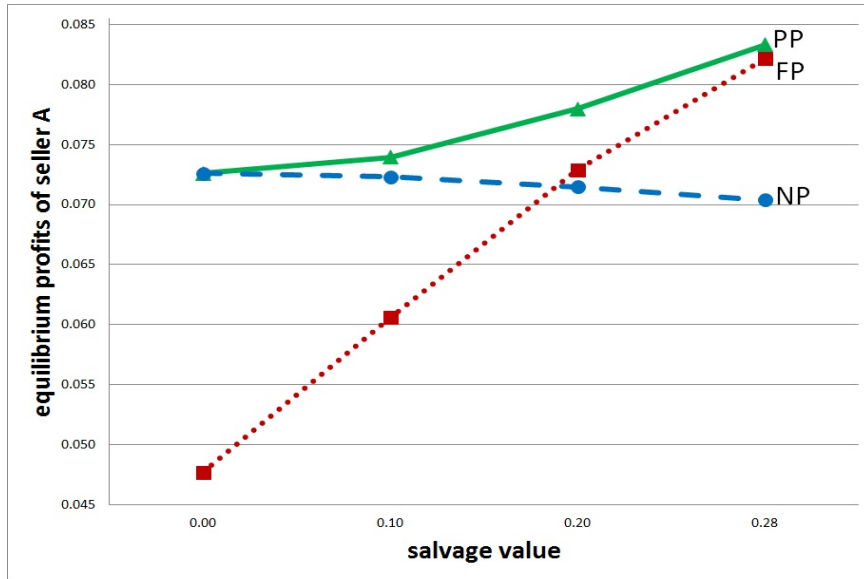


Figure 5.12: seller A's profits when seller B offers partial-refund, $L = 0.3, c = 0.1$

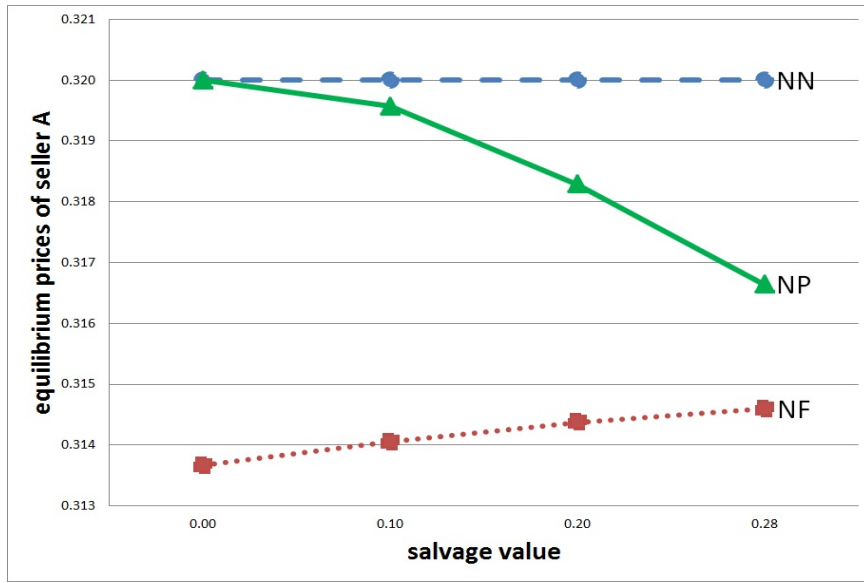


Figure 5.13: seller A's price when he offers no-refund, $L = 0.3, c = 0.1$

same policy. In other words, if both sellers choose not to offer any refund, each of them obtains the highest possible profit.

Similarly, if seller A offers full- or partial-refund, his highest profit occurs when seller B does not offer any refund. In these cases, seller A increases his market size by increasing the customers' willingness to buy.

Consumer surplus and social welfare:

We now explore how social welfare is affected by a return policy. Improving a return policy encourages consumers to purchase the product by increasing their expected valuations. The retailers' total profits, however, may suffer due to a high return rate under a better return policy. To examine the effect of the return policies, we compute and compare the expected social welfare under three symmetric and three asymmetric policies: NN, FF, PP, NF, PF, PN. The expected social welfare is the sum of the expected con-

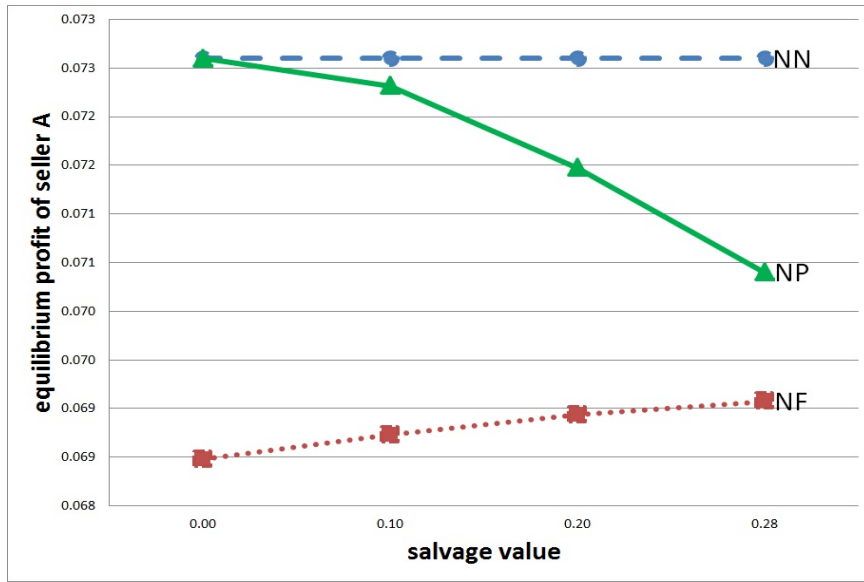


Figure 5.14: seller A's profit when he offers no-refund, $L = 0.3, c = 0.1$

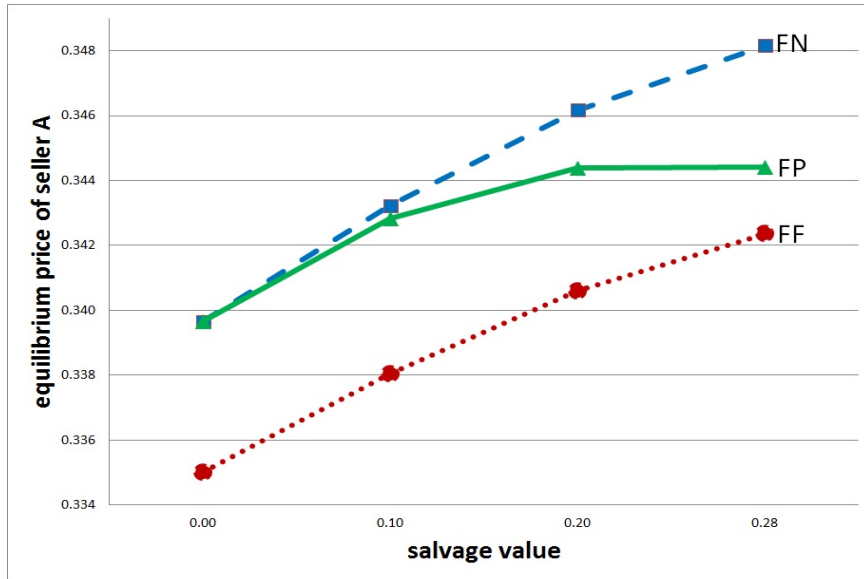


Figure 5.15: seller A's price when he offers full-refund, $L = 0.3, c = 0.1$

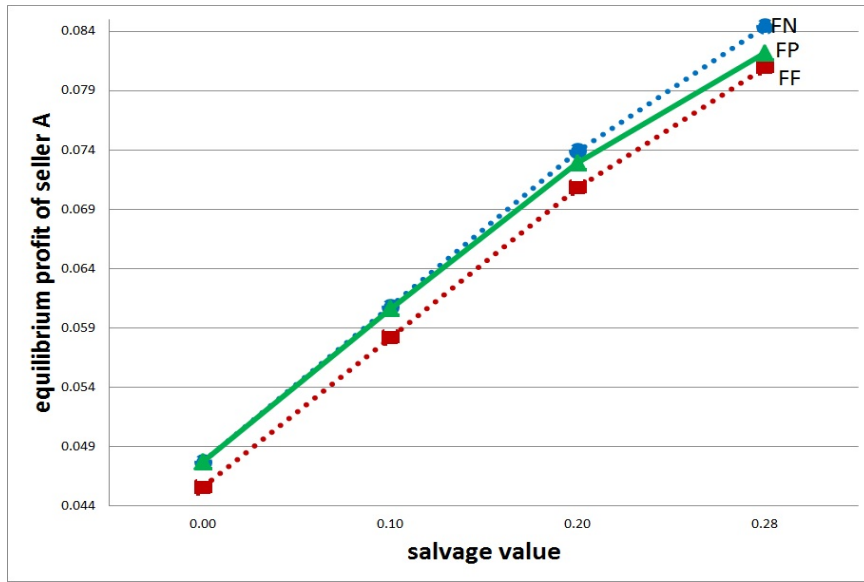


Figure 5.16: seller A's profit when he offers full-refund, $L = 0.3, c = 0.1$

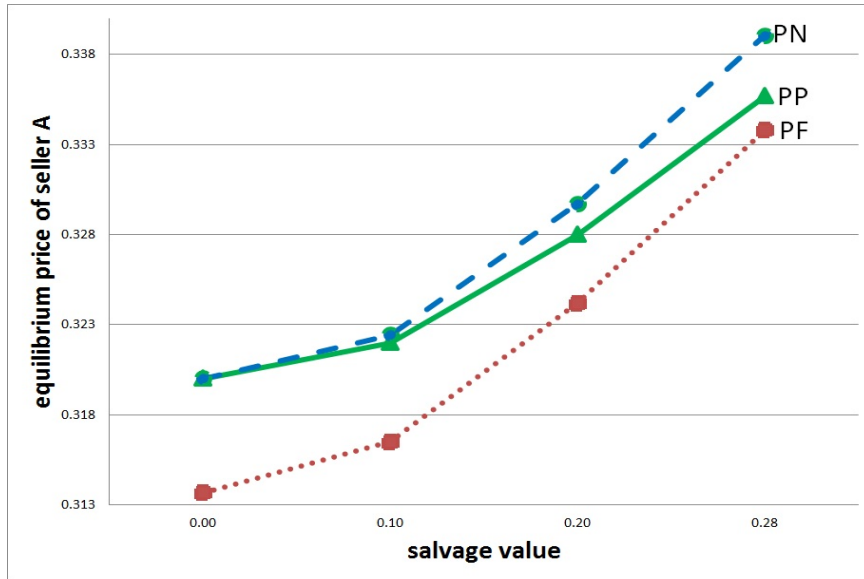


Figure 5.17: seller A's price when he offers partial-refund, $L = 0.3, c = 0.1$

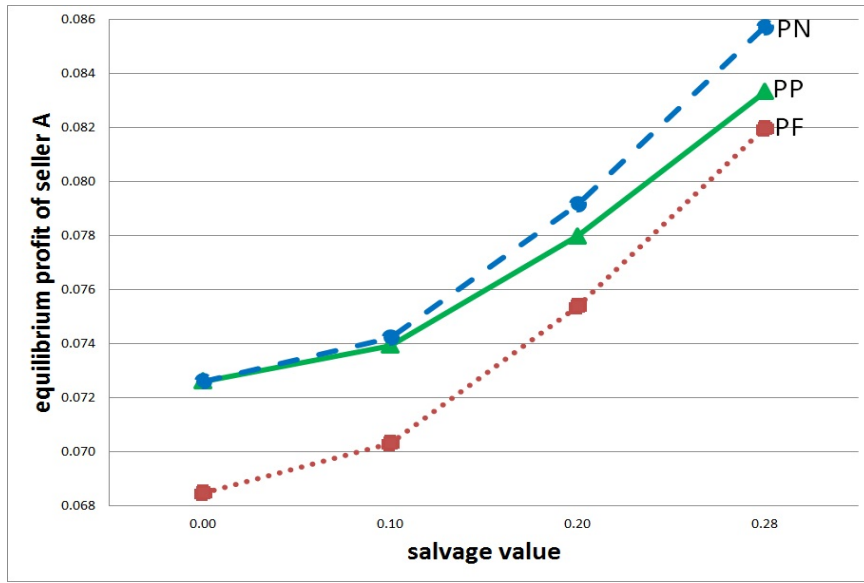


Figure 5.18: seller A's profit when he offers partial-refund, $L = 0.3, c = 0.1$

sumer surplus(i.e., the difference between the total amount that consumers are willing to pay for the product and the total amount that they actually pay) and both sellers' aggregate profit. Table 5.1 shows the consumer surplus and social welfare under different levels of salvage value. As Table 5.1 indicates, although the consumer surplus decreases with salvage value when both sellers offer full-refund policies (FF), it is higher than the other policy combinations for a given salvage value. This is intuitive, since customers do not undertake any risk with a full-refund offer. Under this policy, the reason for a decreasing consumer surplus with salvage value is that a higher price, as a result of a higher salvage, decreases the total market size of a retailer, although the overall effect of the salvage value on the retailer's profit is positive. On the other hand, with respect to social welfare, it can be seen that offering a full-refund is socially efficient, compared to other policies, if the salvage value is high. If the salvage value is low, then offering partial-refunds yields a higher level of social welfare.

Effect of production cost on seller A's perspective:

From seller A's perspective, as the unit production cost increases, the value of the equilibrium price increases, while the equilibrium profit decreases due to a decrease in his market size, as well as unit profit. On the other hand, a change in production cost does not change behavior of either equilibrium prices or profits with respect to salvage value and the level of competition. In other words, comparison between different return policies are not affected by the production cost.

5.6. Summary of Findings

We study consumer returns in a market where two competing retailers sell a homogeneous product to end consumers. These retailers simultaneously decide on the purchase price and choose an appropriate return policy. With the proposed model, we investigate the effects of the sellers' decisions on their respective prices and profits. Our major findings are summarized below:

- A partial-refund policy yields the highest profit for a retailer, regardless of his competitor's return policy. With partial-refunds, the positive effect of such a policy on market size outweighs its negative effect on return cost.
- Increasing salvage value increases profit, since it allows a retailer to improve his refund policy. However, if the seller decides to offer no-refunds, the salvage value has a negative impact on his profit when his competitor offers partial-refunds. The

reason for this observation is that the retailer uses the price to attract customers since he does not do that with a return policy.

- Eliminating the partial-refund option causes a situation of Prisoner's Dilemma. Depending on the salvage value, both retailers may choose to offer full-refund options, though both offering no-refunds yield a higher profit for each retailer.
- Offering full-refunds by both retailers yields higher consumer surplus, while the underlying policy combination is socially efficient only when the salvage value is high. When the salvage value is low, a partial-refund policy improves social welfare.

Table 5.1: Consumer Surplus and Social Welfare

Salvage value	Consumer Surplus					
	NN	FF	PP	NF	PF	PN
0.00	0.064	0.093	0.064	0.079	0.079	0.064
0.10	0.064	0.091	0.066	0.078	0.079	0.065
0.20	0.064	0.090	0.072	0.077	0.081	0.068
0.28	0.064	0.089	0.080	0.076	0.085	0.072
0.30	0.064	0.089	0.082	0.076	0.086	0.073
Salvage value	Social Welfare					
	NN	FF	PP	NF	PF	PN
0.00	0.209	0.184	0.209	0.195	0.195	0.209
0.10	0.209	0.208	0.214	0.207	0.210	0.211
0.20	0.209	0.232	0.228	0.220	0.229	0.219
0.28	0.209	0.251	0.247	0.230	0.249	0.228
0.30	0.209	0.256	0.252	0.232	0.254	0.231

Chapter 6

Conclusion

This dissertation focuses on customer return policies which are commonly used as after sale service. Consumer returns are offered by a retailer in order to increase sales, improve customer satisfaction and diminish customer fit uncertainty. We propose three models, each examining consumer returns from the retailer's perspective and investigate how different market environments affect a retailer's operational and economical decisions in the presence of return policies.

We start our analysis with a basic model when a retailer sells multiple products in a single selling period. In this model, we allow a customer to substitute a variant with a different one, in case the particular variant that a customer desires is looking for is out-of-stock. We assume that each item that is sold may be returned to the retailer with the same probability. Returned items can be brought up to *as-good-as-new* condition after a minor operation. Thus, the retailer can sell them in the same selling

period after testing and repackaging. We investigate the impact of resalable returns and substitution between variants on the optimal order quantities of variants and the total expected profit. With this model discussed in Chapter 3, we show that the optimal order quantity of each variant is affected by the substitution effect, substitutable variants' price and net demand function. Our computational experience shows that as the probability of resalable returns increases, the optimal order quantities decrease since the expected demand that can be satisfied from products which are sold, but not returned decreases as the probability of resalable returns increases. On the other hand, the price of net demand increases as either the return probability or resale probability increases. The total profit also declines as a result of increase in either one of these probabilities. In addition, we show that the optimal order quantity decreases as the probability of customer willingness to switch their unmet demand increases.

The first model mentioned above considers the seller's inventory problem of multiple variants, given that the prices are exogenously determined and the return probability for each product is constant. In the following model, in Chapter 4, we relax these two assumptions and investigate return policies under a modified setting. Our new model assumes that the retailer sells a single unused product to end consumers who face uncertainty in their valuations of the product until they experience it. When a consumer has a return option, she keeps the product only if she has a positive utility after the experiences the product. We assume that consumers exhibit loss-aversion behavior, such that they strongly prefer avoiding losses to realizing gains, i.e. the customers are more sensitive to losses than gains. The seller, aiming to find the optimal price and order

quantity, has an option to offer a refund for returns which is less than the purchase price instead of a refund equal to the purchase price. With this proposed model, we find that the seller needs to lower the purchase price to compensate for the consumers' sensitivity, even if he offers a high refund for a return. In addition, the seller has to offer more generous refunds if he chooses to offer a partial-refund. As the loss-aversion degree increases, the optimal refund amount increases and approaches the purchase price. Thus, the profit decreases as the degree of loss-aversion goes up. Moreover, loss-aversion leads the seller to carry lower fewer inventories, which decreases the customer service level.

In the final model presented in Chapter 5, we analyze consumer returns in a competitive market, where two sellers simultaneously decide their prices and return policies in order to increase their market sizes. The market share of each seller is affected by his and the rival's prices and their return policies, customer's random valuations of the product and the degree of competition between the firms. Our analysis reveals that the equilibrium return policies and prices depend on a number of factors, such as the product's salvage value and the level of competition. We show that a partial refund policy is in equilibrium under our competitive scenario. On the other hand, consumers are better off if both firms offer full refund policies. While the underlying combination is socially efficient when the salvage value is high, a partial-refund policy offered by each seller improves social welfare when the salvage value is low.

In the models that suggested in this study, we observe that the return policies improve as the retailer has a better option to salvage the product. i.e. larger salvage value

except in the situation where the retailer offers the no-refund policy while his rival offers a partial-refund policy. On the other hand, better return policies do not guarantee a higher profit for the retailer. If the retailer has an option to offer a partial-refund, it yields the highest profit compared to the full- and no-refund cases although the partial-refund is not the most favorable policy by customers. The full-refund policy, since the customers has no risk when it is offered, always provides the highest consumer surplus. However, although the full-refund policy yields the highest consumer surplus, it is socially efficient only when the product's salvage value is high. Otherwise, the partial-refund policy yields a higher level of social welfare.

There are some limitations that our research may have. Those limitations yield good directions to extent this study for future research. For example, we assume that there are two retailers competing for market size in our third model discussed in Chapter 5. It may be good to extend our model to the market scenario when there are more than two sellers. In such a scenario, the equilibrium return policies may depend on the product's salvage value and the degree of competition degree between each pair of retailers. Since a seller's problem is to choose appropriate return policy and to find the optimal price, the equilibrium return policies are likely be similar to those under duopoly.

Another limitation of our third model is that both retailers have enough capacity to cover the entire market and the demand function of each seller depends on the respective purchase prices and return policy decisions (e.g. demand is a linear function of price). While a linear function often facilitates tractable analysis, it may oversimplify the real

world. A possible extension to this model is to relax the unlimited capacity assumption and to analyze the inventory problem of competing retailers. Furthermore, future exploration may incorporate random demand. We assume in Chapter 5 that a consumer incurs a disutility when purchasing a product. However, returning a product in case of a misfit also creates an additional cost for the consumer as well as for the firm. Thus, the incorporation of a return processing cost would be another extension.

There are other possible extensions of the models proposed in this study for advancing the knowledge base in this one of research. Considering a supply chain where consumers have an option to return in case of a product misfit may be a fruitful future research direction. In the literature, there exist studies that investigate coordination of supply chains consisting of a manufacturer and a retailer facing consumer returns. Some of such studies assume *ex ante* homogeneous customers who face valuation uncertainty before purchase (Su (2009), Xiao et al. (2010)), while others consider reverse logistics structures with the assumption of exogenous return processes (Fleischmann et al. (2002), DeCroix et al. (2005)). To the best of our knowledge, there are no studies which investigate the effect of the manufacturer's contract type on the performance of the supply chain where a manufacturer supplies a product to competitive retailers who choose their own retail prices and product return policies.

In each model that we suggest assume an exogenous salvage value. However, there exist cases where the retailer has right to choose an appropriate salvage value for return products. For example, products such as apparel and electronics are sometimes sold

with price markdowns by the retailer. In such cases, the retailer may prefer to find the optimal salvage value which maximizes his total profit along with other decision variables. So, assuming endogenous salvage value is another research extension that need to be investigated.

Future research may also focus on how the performance of returns policies changes if a retailer has tools to reduce uncertainties concerning product utility and fit. One way to reduce such uncertainty is to better inform the consumer before purchase, or offer a trial period so that the consumer has less hesitation to purchase the product. Ignoring the impact of loss-averse customers in a monopolistic and competitive markets are other possible extensions that can be explored. In addition, future researchers may focus on multi-period models. Under a multi-period setting, consumers may be allowed to make multiple purchases from the same firm or to switch firms over time.

It is our sincere hope that this study has contributed to the existing literature of consumer returns that may be of interest to researchers in the disciplines of operations management, marketing and economics, by providing some insights into regarding the impact of consumer returns on retailers operational and economical decisions. We also hope that this work will form the basis for meaningful future research in the above mentioned disciplines.

Appendix

The Effect of Substitution and Resalable Return on Retailer's Inventory Decision

Proof of the Optimal Order Quantity: The optimal order quantity of variant i that maximizes the expected profit in equation 3.3 is equivalent to minimizing

$$E[(N_i - q_i)^+] \{ \alpha_{ii}(p_i^N + s_i^N - v) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij} \} + (c_i - v)q_i. \quad (6.1)$$

The optimal quantity can be calculated from the first order optimality condition of the above expression, which is

$$\frac{\partial TP(q_i)}{\partial q_i} = -(1 - F(q_i)) \{ \alpha_{ii}(p_i^N + s_i^N - v) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij} \} + (c_i - v_i) \quad (6.2)$$

From the first order condition above, the optimal order quantity for variant i is:

$$q_i^* = F_i^{-1} \left(\frac{\alpha_{ii}(p_i^N + s_i^N - v) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij} - c_i + v}{\alpha_{ii}(p_i^N + s_i^N - v) - \sum_{j=1}^{i-1} p_j^N \alpha_{ij}} \right) \quad (6.3)$$

Since equation 3.3 is strictly concave with respect to q_i , the quantities found by 6.3 are global optimal values. \square

Effect of r and k on p_i^N :

$$p_i^N = p_i^G[1 + rk + (rk)^2 + \dots] = p_i^G/(1 - rk); \quad (6.4)$$

$$\frac{\partial p_i^N}{\partial r} = -\frac{(1 - k)(p_i - v) + d}{(1 - rk)^2}; \quad (6.5)$$

$$\frac{\partial p_i^N}{\partial k} = -\frac{r((1 - r)(v - p_i) + rd)}{(1 - rk)^2}; \quad (6.6)$$

$\frac{\partial p_i^N}{\partial r} \leq 0$ since $(p_i - v)$ is always greater than zero. On the other hand, $\frac{\partial p_i^N}{\partial k} \geq 0$ when $r \leq \frac{p_i - v}{p_i - v + d}$. Observe that if d is not too large, $\frac{\partial p_i^N}{\partial k} \geq 0$ for almost all r . \square

Pricing and Customer Returns Policies with Loss Averse Customers

Proof of Theorem 4.3.1: The seller maximizes $\pi^N(p, q; r = 0)$ subject to $EU^N(p; r = 0) \geq 0$. Note that EU^N is independent of q . Since π^N increases in p , the seller should choose the largest p that makes $EU^N(p; r = 0) \geq 0$, and then choose an optimizing q .

$$\frac{\partial EU^N(p; r = 0)}{\partial p} = -1 + (\lambda - 1)(-G(p)) < 0.$$

At the two end points, if $p = 0$, $EU^N = Ev > 0$, and if $p = Ev$, $EU^N = (\lambda - 1) \int_0^{Ev} (v - Ev)dG(v) < 0$. Thus there exists a unique solution $p_\lambda^N \in (0, Ev)$ such that $EU^N \geq 0$ if

and only if $p \leq p_\lambda^N$. It is then optimal to charge p_λ^N and the optimal q is obtained by standard newsvendor logic. \square

Proof of Lemma 4.3.2: Using the implicit function theorem on $EU^N(p; r = 0)$, we have

$$-\frac{\partial p_\lambda^N}{\partial \lambda}(1 + (\lambda - 1)G(p_\lambda^N)) + \int_0^{p_\lambda^N} (v - p_\lambda^N)dG(v) = 0.$$

Since $\int_0^{p_\lambda^N} (v - p_\lambda^N)dG(v) \leq 0$ and $1 + (\lambda - 1)G(p_\lambda^N) > 0$, $\frac{\partial p_\lambda^N}{\partial \lambda} \leq 0$. Similarly, for $c \geq s$,

$$\frac{\partial q_\lambda^N}{\partial \lambda} = \frac{\partial p_\lambda^N}{\partial \lambda} \frac{c - s}{(p_\lambda^N - s)^2 f(q_\lambda^N)} \leq 0.$$

\square

Proof of Lemma 4.3.4: The profit of full-refund policy is independent of λ , whereas the optimal profit under the no-refund policy monotonically decreases, i.e.

$$\begin{aligned} \frac{\partial \Pi^F}{\partial \lambda} &= 0, \\ \frac{\partial \Pi^N}{\partial \lambda} &= \frac{\partial p_\lambda^N}{\partial \lambda} E \min(X, q_\lambda^N) + \frac{\partial q_\lambda^N}{\partial \lambda} ((p_\lambda^N - s)\bar{F}(q_\lambda^N) - (c - s)) = \frac{\partial p_\lambda^N}{\partial \lambda} E \min(X, q_\lambda^N) \leq 0. \end{aligned}$$

For any given s and c , if $\pi^F \geq \pi^N$ when $\lambda = 1$, then $\pi^F \geq \pi^N$ for any $\lambda > 1$. Thus, $\bar{\lambda} = 1$. Otherwise since π^F is constant in λ and π^N decreases, there exists a threshold

$\bar{\lambda} > 1$ such that $\pi^F \geq \pi^N$, for $\lambda \geq \bar{\lambda}$. \square

Proof of Lemma 4.3.5: If $\bar{\lambda} > 1$, then we can show that $\bar{\lambda}$ is the solution of $H(\lambda, s) = (p^F - s)\bar{G}(p^F) - (p_\lambda^N - s) = 0$, since if this is the case, by Theorems 4.3.1 and 4.3.3 we have $q^F = q_\lambda^N$, and thus $\pi^F = \pi^N$. As p^F is independent of λ and p_λ^N decreases in λ , $H(\lambda, s)$ increases in λ , so that there is a unique solution $\bar{\lambda}$. For a given λ , $H(\lambda, s)$ increases with salvage value, i.e. $\frac{\partial H(\lambda, s)}{\partial s} = -\bar{G}(p^F) + 1 \geq 0$. Therefore, $\bar{\lambda}$ should decrease in s .

On the other hand, since $(p^F - s)\bar{G}(p^F)$ and p_λ^N are independent of production cost c , $\bar{\lambda}$ does not change with c . \square

Proof of Theorem 4.3.6: First, $\frac{EU^P(p, r)}{\partial p} = -1 + (\lambda - 1)(-G(p)) < 0$, so there exists a unique $p^P(r)$ such that $EU^P(p, q, r) \geq 0$ for $p \leq p^P(r)$. Since $\pi^P(p, q, r)$ increases in p , the firm should charge $p = p^P(r)$ so that $EU^P(p, q, r) = 0$.

For a given q , the seller chooses r to maximize π^P subject to $EU^P(p^P(r), r) = 0$. Note that in π^P , only the term $(p - s)\bar{G}(r) + (p - r)G(r)$ depends on r , so the problem is equivalent to:

$$\begin{aligned} \max_r \quad & (p - s)\bar{G}(r) + (p - r)G(r) \\ \text{s.t.} \quad & EU^P(p, q, r) = Ev - p + (\lambda - 1) \int_0^p (v - p)dG(v) + \lambda \int_0^r (r - v)dG(v) = 0. \end{aligned}$$

Using a Lagrangian multiplier ξ ,

$$L(p, r, \xi) = (p-s)\bar{G}(r) + (p-r)G(r) + \xi \left[Ev - p + (\lambda - 1) \int_0^p (v - p) dG(v) + \lambda \int_0^r (r - v) dG(v) \right],$$

and at the optimal solution we must have $\frac{\partial L}{\partial p} = \frac{\partial L}{\partial r} = \frac{\partial L}{\partial \xi} = 0$:

$$\frac{\partial L}{\partial p} = 1 + \xi[-1 + (\lambda - 1)(-G(p))] = 0, \quad (6.7)$$

$$\frac{\partial L}{\partial r} = (s - r)g(r) - G(r) + \xi \lambda G(r) = 0. \quad (6.8)$$

Substituting (6.7) into (6.8), we have

$$\lambda G(r) - [(r - s)g(r) + G(r)][1 + (\lambda - 1)G(p)] = 0. \quad (6.9)$$

Therefore, the optimal p_λ^P and r_λ^P are the solutions to (6.9) and $EU^P = 0$. q_λ^P is then solved by standard Newsvendor logic. \square

Proof of Lemma 4.3.7: (6.9) is equivalent to $\frac{\lambda}{1 + (\lambda - 1)G(p)} = (r - s)\frac{g(r)}{G(r)} + 1$.

Since $LHS - 1 = \frac{(\lambda - 1)(1 - G(p))}{1 + (\lambda - 1)G(p)} \geq 0$, $RHS - 1 = (r - s)\frac{g(r)}{G(r)}$ needs to be nonnegative, implying $r \geq s$ at optimality. \square

Proof of Lemma 4.3.8: (1) For a fixed p , since $E\pi^P$ decreases in r and EU^P is monotone in r , the seller should choose $r^*(p)$ such that $EU^P = \int_p^\infty (v - p) dG(v) + \lambda \int_r^p (v - p) dG(v) + \lambda \int_0^r (r - p) dG(v) = 0$. Applying implicit theorem on EU^P , we have

$$\int_0^p (v - p) dG(v) + \int_0^{r^*(p)} (r^*(p) - v) dG(v) + \lambda G(r^*(p)) \frac{\partial r^*(p)}{\partial \lambda} = 0, \text{ therefore } \frac{\partial r^*(p)}{\partial \lambda} \geq 0.$$

For quantity,

$$\frac{\partial q^*(p)}{\partial \lambda} = \frac{\partial r^*(p)}{\partial \lambda} \frac{(c - s)[G(r^*(p)) + (r^*(p) + p - s)g(r^*(p))]}{[(p - r^*(p))G(r^*(p)) + (p - s)\bar{G}(r^*(p))]^2 f(q^*(p))} \geq 0.$$

(2) For a fixed r , since $E\pi^P$ increases in p and EU^P is monotone in p , $p^*(r)$ is the solution of $EU^P = \int_p^\infty (v - p) dG(v) + \lambda \int_r^p (v - p) dG(v) + \lambda \int_0^r (r - p) dG(v) = 0$. Again,

$$\text{the implicit theorem, } -\frac{\partial p^*(r)}{\partial \lambda} [1 + (\lambda - 1)G(p^*(r))] + \int_r^{p^*(r)} (v - p^*(r)) dG(v) + \int_0^r (r - p^*(r)) dG(v) = 0. \text{ So, } \frac{\partial p^*(r)}{\partial \lambda} \leq 0. \text{ Similarly,}$$

$$\frac{\partial q^*(r)}{\partial \lambda} = \frac{\partial p^*(r)}{\partial \lambda} \frac{(c - s)}{[(p^*(r) - r)G(r) + (p^*(r) - s)\bar{G}(r)]^2 f(q^*(r))} \leq 0.$$

□

Proof of Lemma 4.3.9: Given the same p , $EU^N(p; r = 0) \leq EU^P(p, r) \leq EU^F(p; r = p)$, and EU^N, EU^P, EU^F all decrease in p , the optimal roots also follow the same order.

Note that a partial refund policy, if $r > p$, setting $r = p$ still generates positive expected utilities for consumers, but increases the sellers' profit. Thus the optimal $r_\lambda^P \leq p_\lambda^P$. Since $r^N = 0$, we have $r^N \leq r_\lambda^P \leq p_\lambda^P \leq p^F = r^F$. □

Uniform distribution under partial refund policy : Assume that the con-

sumers' valuations are uniformly distributed in $[0, 1]$. Then $EU^P = 0$ becomes $p = \frac{-1 + \sqrt{\lambda + \lambda^2 r^2 - \lambda r^2}}{\lambda - 1}$. Combining this with (6.9), we can obtain the following inverse function of r_λ^P : $\lambda = \frac{(2r - s)^2(r^2 - 1)}{r^2[(2r - s)^2 - 1]}$. Note that the RHS being positive implies that $2r - s < 1$, and it can be verified that the RHS is increasing in r for $r \in [s, \frac{1+s}{2}]$, thus r_λ^P increases in λ . Similarly, $EU^P = 0$ can be written as $r = \frac{\sqrt{\lambda(-1 + 2p + p^2\lambda - p^2)}}{\lambda}$. Combining this with (6.9) and solving for p_λ^P yield a cumbersome expression which is omitted here, but it can be verified that it increases with λ . \square

Pricing and Customer Returns Policies for Two Competitive Retailers

Proof of 5.3.2: The profit function of retailer A for Case 2:

Case 2.1 If $Ev - p_A > \frac{1}{2}(L + p_B - p_A) \Leftrightarrow p_A < 2Ev - L - p_B$, then

$$\begin{aligned}\Pi_A^1 &= (p_A - c_A)(Ev - p_A + \frac{1}{2}(L + p_B - p_A)), \\ \frac{\partial \Pi_A^1}{\partial p_A} &= Ev + \frac{1}{2}L + \frac{1}{2}p_B + \frac{3}{2}c_A - 3p_A.\end{aligned}$$

Case 2.2 If $Ev - p_A < \frac{1}{2}(L + p_B - p_A) \Leftrightarrow p_A > 2Ev - L - p_B$, then

$$\begin{aligned}\Pi_A^2 &= (p_A - c_A)2(Ev - p_A), \\ \frac{\partial \Pi_A^2}{\partial p_A} &= 2Ev + 2c_A - 4p_A.\end{aligned}$$

At $p_A = 2Ev - L - p_B$, $\Pi_A^1 = \Pi_A^2$. In addition, the difference between the first derivatives at $p_A = 2Ev - L - p_B$ is as follows:

$$\begin{aligned} \left(\frac{\partial \Pi_A^1}{\partial p_A} - \frac{\partial \Pi_A^2}{\partial p_A} \right) \Big|_{(2Ev-L-p_B)} &= \frac{1}{2}(2Ev - L - p_B - c_A) = \frac{1}{2}(p_A - c_A) > 0 \\ \Rightarrow \frac{\partial \Pi_A^1}{\partial p_A}(2Ev - L - p_B) &> \frac{\partial \Pi_A^2}{\partial p_A}(2Ev - L - p_B) \end{aligned}$$

Thus, the optimal response function of retailer A is piecewise function.

1. If $\frac{\partial \Pi_A^1}{\partial p_A}(2Ev - L - p_B) = -5Ev + \frac{7}{2}L + \frac{7}{2}p_B + \frac{3}{2}c_A \leq 0$, then $p_A^*(p_B) = \frac{1}{3}(Ev + \frac{1}{2}L + \frac{1}{2}p_B + \frac{3}{2}c_A)$.
2. If $\frac{\partial \Pi_A^1}{\partial p_A}(2Ev - L - p_B) = -5Ev + \frac{7}{2}L + \frac{7}{2}p_B + \frac{3}{2}c_A > 0$, and $\frac{\partial \Pi_A^2}{\partial p_A}(2Ev - L - p_B) = -6Ev + 4L + 4p_B + 2c_A \leq 0$, then $p_A^*(p_B) = 2Ev - L - p_B$.
3. If $\frac{\partial \Pi_A^2}{\partial p_A}(2Ev - L - p_B) = -6Ev + 4L + 4p_B + 2c_A > 0$, then $p_A^*(p_B) = \frac{1}{2}(Ev + c_A)$.

□

Proof of Proposition 5.3.3: The response functions of retailer A and B intersect at $p_A^e = p_B^e = \frac{1}{5}(2Ev + L + 3c_A)$ if $-5Ev + \frac{7L+7p_B-3c_A}{2} < 0$, in other words, if $L < \frac{6}{7}(Ev - c_A)$.

The response functions of retailer A and B intersect at $p_A^e = p_B^e = \frac{1}{2}(Ev + c_A)$ if $-6Ev + 4L + 4p_B + 2c_A > 0$ or if $L > Ev - c_A$.

Next, if $-5Ev + \frac{7L+7p_B-3c_A}{2} > 0$ and $-6Ev + 4L + 4p_B + 2c_A < 0$, in other words $\frac{6}{7}(Ev - c_A) < L < Ev - c_A$, equilibrium prices p_A^e and p_B^e are any combination of points on $p_A + p_B = Ev - c_A$.

□

Proof of Lemma 5.3.4: First, we prove that $p^*(p_B) \leq \bar{p}_A$.

$$\frac{\partial \Pi_A^1}{\partial p_A} = (\bar{G}(p_A) + (s_A - p_A)g(p_A))(Emax(v, p_A) - p_A + \frac{1}{2}(Emax(v, p_A) - Emax(v, p_B) + p_B - p_A + L)) + (p_A - c_A + (s_A - p_A)G(p_A))(-\frac{3}{2}\bar{G}(p_A))$$

At optimal p_A , the net unit profit $p_A - c_A + (s_A - p_A)G(p_A) \geq 0$ and the market size $(Emax(v, p_A) - p_A + \frac{1}{2}(Emax(v, p_A) - Emax(v, p_B) + p_B - p_A + L)) \geq 0$. On the other hand, for $p_A > \bar{p}_A$, $\bar{G}(p_A) + (s_A - p_A)g(p_A) < 0$. Thus, any $p_A > \bar{p}_A$ cannot be optimal since $\frac{\partial \Pi_A^1}{\partial p_A} < 0$.

Similarly, $\frac{\partial \Pi_A^2}{\partial p_A} = (\bar{G}(p_A) + (s_A - p_A)g(p_A))2(Emax(v, p_A) - p_A) + (p_A - c_A + (s_A - p_A)G(p_A))(-2\bar{G}(p_A))$. For any $p_A > \bar{p}_A$, $\frac{\partial \Pi_A^2}{\partial p_A} < 0$. Therefore, the optimal $p^*(p_B) \leq \bar{p}_A$.

Next, we show that Π_A is unimodal in p_A . It suffices to show that both Π_A^1 and Π_A^2 are unimodal and $\frac{\partial \Pi_A^1}{\partial p_A} \geq \frac{\partial \Pi_A^2}{\partial p_A}$ at $p_A = \tilde{p}_A(p_B)$. We can rewrite the derivative of Π_A^1 as follows;

$$\frac{\partial \Pi_A^1}{\partial p_A} = \bar{G}(p_A)[(1 + (s_A - p_A)\frac{g(p_A)}{\bar{G}(p_A)})(Emax(v, p_A) - p_A + \frac{1}{2}(Emax(v, p_A) - Emax(v, p_B) + p_B - p_A + L)) - \frac{3}{2}(p_A - c_A + (s_A - p_A)G(p_A))] = \bar{G}(p_A)H_A^1(p_A, p_B).$$

If H_A^1 is a monotonic function, then Π_A^1 is unimodal.

$$\frac{\partial H_A^1}{\partial p_A} = (-\frac{g(p_A)}{\bar{G}(p_A)} + (s_A - p_A)\frac{\partial}{\partial p_A}(\frac{g(p_A)}{\bar{G}(p_A)}))(Emax(v, p_A) - p_A + \frac{1}{2}(Emax(v, p_A) - Emax(v, p_B) + p_B - p_A + L)) + (1 + (s_A - p_A)\frac{g(p_A)}{\bar{G}(p_A)})(-\frac{3}{2}\bar{G}(p_A)) - \frac{3}{2}(\bar{G}(p_A) + (s_A - p_A)g(p_A)).$$

We know that for $p_A \leq \bar{p}_A$, $\bar{G}(p_A) + (s_A - p_A)g(p_A) > 0$. In addition, $s_A < p_A$ and $\frac{g(p_A)}{\bar{G}(p_A)}$ increases in p_A since G has increasing failure rate (IFR). So, $\frac{\partial H_A^1}{\partial p_A} < 0$ and this result shows that Π_A^1 is unimodal. Similarly, we can show that Π_A^2 is unimodal.

Finally, we check the difference of first order derivatives at \tilde{p}_A .

$$\left(\frac{\partial \Pi_A^1}{\partial p_A} - \frac{\partial \Pi_A^2}{\partial p_A} \right) \Big|_{\tilde{p}_A} = \frac{1}{2} \overline{G}(\tilde{p}_A) [\tilde{p}_A - c_A + (s_A - \tilde{p}_A) \overline{G}(\tilde{p}_A)] \geq 0$$

Therefore, Π_A is unimodal in p_A for $p_A \leq \bar{p}_A$. \square

Proof of Lemma 5.3.5: $\frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) = \overline{G}(\tilde{p}_A) [(1 + (s_A - \tilde{p}_A) \frac{g(\tilde{p}_A)}{\overline{G}(\tilde{p}_A)}) 2(Emax(v, \tilde{p}_A) - \tilde{p}_A) - \frac{3}{2}(\tilde{p}_A - c_A + (s_A - \tilde{p}_A) \overline{G}(\tilde{p}_A))] = \overline{G}(\tilde{p}_A) \tilde{H}_A^1(\tilde{p}_A).$

Since $\tilde{H}_A^1(\tilde{p}_A)$ decreases in \tilde{p}_A , let $\underline{\tilde{p}}_A$ be the unique solution such that $\tilde{H}_A^1(\underline{\tilde{p}}_A) = 0$. If G is IFR, there exists a unique solution \underline{p}_B such that $\underline{\tilde{p}}_A = Emax(v, \underline{\tilde{p}}_A) + Emax(v, \underline{p}_B) - \underline{p}_B - L$. For any $p_B < \underline{p}_B$, $\tilde{p}_A(p_B) > \underline{\tilde{p}}_A$ and $\tilde{H}_A^1(\tilde{p}_A(p_B)) < 0$. Thus, $\frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) < 0$ which yields the optimal solution $p_A^*(p_B) = argmax \Pi_A^1(p_A, p_B)$.

For $p_B > \underline{p}_B$, $\frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) > 0$.

$\frac{\partial \Pi_A^2}{\partial p_A}(\tilde{p}_A) = \overline{G}(\tilde{p}_A) [(1 + (s_A - \tilde{p}_A) \frac{g(\tilde{p}_A)}{\overline{G}(\tilde{p}_A)}) 2(Emax(v, \tilde{p}_A) - \tilde{p}_A) - 2(\tilde{p}_A - c_A + (s_A - \tilde{p}_A) \overline{G}(\tilde{p}_A))] = \overline{G}(\tilde{p}_A) \tilde{H}_A^2(\tilde{p}_A).$

Similar to $\tilde{H}_A^1(\tilde{p}_A)$, $\tilde{H}_A^2(\tilde{p}_A)$ decreases in \tilde{p}_A . Let $\widetilde{\tilde{p}}_A$ be the unique solution such that $\tilde{H}_A^2(\widetilde{\tilde{p}}_A) = 0$. If G is IFR, there exists a unique solution \bar{p}_B such that $\widetilde{\tilde{p}}_A = Emax(v, \widetilde{\tilde{p}}_A) + Emax(v, \bar{p}_B) - \bar{p}_B - L$. Then for $p_B > \bar{p}_B$, $\tilde{p}_A(p_B) < \widetilde{\tilde{p}}_A$ and $\tilde{H}_A^2(\tilde{p}_A(p_B)) > 0$. Thus, $\frac{\partial \Pi_A^2}{\partial p_A}(\tilde{p}_A) > 0$ and the optimal solution is $p_A^*(p_B) = argmax \Pi_A^2(p_A, p_B)$.

Finally, $\underline{p}_B < p_B < \bar{p}_B$, the optimal solution is $p_A^*(p_B) = \tilde{p}_A$. \square

Proof of Proposition 5.3.6: First, we show that (p_A^{1e}, p_B^{1e}) is unique. Apply the chain rule to the FOC of Π_A^1 yields $\frac{\partial p_A^1}{\partial p_B} (-g(p_A^1) H_A^1(p_A^1, p_B^1) + \overline{G}(p_A^1) \frac{\partial H^1}{\partial p_A^1}) + \overline{G}(p_A^1) \frac{\partial H^1}{\partial p_B^1} =$

0. We know that $H_A^1(p_A^1, p_B^1) = 0$ and $\frac{\partial H^1}{\partial p_A^1} < 0$. In addition, $\frac{\partial H^1}{\partial p_B^1} = (1 + (s_A - p_A)\frac{g(p_A)}{\bar{G}(p_A)})\frac{1}{2}\bar{G}(p_B) \geq 0$. Thus, the optimal solution of $\argmax \Pi_A^1(p_A, p_B)$, $p_A^{1*}(p_B)$ increases in p_B .

If $(p_A^{1*} < p_B)$, then $\bar{G}(p_A^{1*}) > \bar{G}(p_B)$, then $\frac{\partial H^1}{\partial p_B^1} < |\frac{\partial H^1}{\partial p_A^1}(p^{1*}(p_B))|$. Thus, $|\frac{\partial p_A^{1*}}{\partial p_B}| < 1$ if $p_A^{1*} \leq p_B$ which means that (p_A^{1e}, p_B^{1e}) is unique.

Since Π_A^2 is independent of p_B and $p_A^{2*}(p_B)$ is a constant, (p_A^{2e}, p_B^{2e}) are unique. Also, $\tilde{p}_A(p_B)$ decreases in p_B .

Since all parameters are symmetric, the equilibrium (p_A^{1e}, p_B^{1e}) is symmetric as well and p_A^{1e} solves

$H_A^1(p_A, p_B) = (1 + (s_A - p_A)\frac{g(p_A)}{\bar{G}(p_A)})(Emax(v, p_A) - p_A + \frac{1}{2}L) - \frac{3}{2}(p_A - c_A + (s_A - p_A)G(p_A)) = 0$. By using the implicit theorem, it can be shown that both p_A^{1e} and p_B^{1e} increase in L . On the other hand, p_A^{1e}, p_B^{1e} should satisfy $p_A^{1e} < Emax(v, p_A^{1e}) + Emax(v, p_B^{1e}) - p_B^{1e} - L$ since Π_A^1 is valid only for Case 2.1. For the symmetric case, this condition is the same as $Emax(v, p_A^{1e}) - p_A^{1e} > \frac{1}{2}L$. Let $K(L) = Emax(v, p_A^{1e}) - p_A^{1e} - \frac{1}{2}L$, then $\frac{\partial K}{\partial L} = -\bar{G}(p_A^{1e})\frac{\partial p_A^{1e}}{\partial L} - \frac{1}{2} < 0$.

Let \underline{L}^{FF} be the unique solution of $K(L) = 0$, then for $L < \underline{L}^{FF}$, $K(L) > 0$ and (p_A^{1e}, p_B^{1e}) is the equilibrium. Again, let $\bar{L}^{FF} = 2(Emax(v, p_A^{2e}) - p_A^{2e})$, then for $L > \bar{L}^{FF}$, (p_A^{2e}, p_B^{2e}) is the Nash equilibrium and for $\underline{L}^{FF} < L < \bar{L}^{FF}$, $(\tilde{p}_A^e, \tilde{p}_B^e)$ is the Nash equilibrium. \square

Proof of Proposition 5.3.7: The proof is omitted since it is similar to the above situations. \square

Proof of Proposition 5.5.1: Let $\tilde{p}_A(p_B, r_B, r_A) = Emax(v, r_A) + Emax(v, r_B) - p_B - L$. Π_A is continuous at \tilde{p}_A . In addition, it can be shown that given (p_B, r_B, r_A) , Π^1 and Π^2 are concave in p_A . The first order derivatives of Π^1 and Π^2 are as follows:

$$\begin{aligned}\frac{\partial \Pi_A^1}{\partial p_A} &= \frac{3}{2}(Emax(v, r_A) - p_A) + \frac{1}{2}(-Emax(v, r_B) + p_B + L) - \frac{3}{2}(p_A - c_A + (s_A - r_A)G(r_A)), \\ \frac{\partial \Pi_A^2}{\partial p_A} &= 2(Emax(v, r_A) - 2p_A + c_A - (s_A - r_A)G(r_A)).\end{aligned}$$

At $\tilde{p}_A(p_B, r_B, r_A)$, Π_A^1 has a larger derivative than Π_A^2 , i.e.

$$\frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) - \frac{\partial \Pi_A^2}{\partial p_A}(\tilde{p}_A) = \frac{1}{2}(p_A - c_A + (s_A - r_A)G(r_A)) > 0.$$

Thus, there are 3 candidates for the equilibrium point:

$$p_A^*(p_B, r_B, r_A) = \begin{cases} p_A^{1*}(p_B, r_B, r_A) &= argmax \Pi_A^1 = \frac{1}{6}(3Emax(v, r_A) - Emax(v, r_B) + p_B + L \\ &\quad + 3c_A - 3(s_A - r_A)G(r_A)), \\ p_A^{2*}(p_B, r_B, r_A) &= argmax \Pi_A^2 = \frac{1}{2}(Emax(v, r_A) + c_A - (s_A - r_A)G(r_A)), \\ \tilde{p}_A &= Emax(v, r_A) - Emax(v, r_B) + p_B - L \end{cases}$$

If $p_A^*(p_B, r_B, r_A) = p_A^{1*}$, then

$$\begin{aligned}\Pi_A &= \frac{1}{36}(3Emax(v, r_A) - Emax(v, r_B) + p_B + L - 3c_A - 3(s_A - r_A)G(r_A))^2, \\ \frac{\partial \Pi_A}{\partial r_A} &= \frac{1}{18}(3Emax(v, r_A) - Emax(v, r_B) + p_B + L - 3c_A - 3(s_A - r_A)G(r_A))(s_A - r_A)g(r_A).\end{aligned}$$

Π_A is maximized at $r_A = s_A$.

If $p_A^*(p_B, r_B, r_A) = \tilde{p}_A$, then

$$\begin{aligned}\Pi_A &= (Emax(v, r_A) + Emax(v, r_B) - p_B - L - c_A + (s_A - r_A)G(r_A))2(-Emax(v, r_B) + p_B + L), \\ \frac{\partial \Pi_A}{\partial r_A} &= 2(s_A - r_A)g(r_A)(-Emax(v, r_B) + p_B + L)\end{aligned}$$

Thus, Π_A is maximized at $r_A = s_A$.

If $p_A^*(p_B, r_B, r_A) = p_A^{2*}$, then

$$\begin{aligned}\Pi_A &= \frac{1}{2}(Emax(v, r_A) - c_A + (s_A - r_A)G(r_A))^2, \\ \frac{\partial \Pi_A}{\partial r_A} &= (Emax(v, r_A) - c_A + (s_A - r_A)G(r_A))(s_A - r_A)g(r_A).\end{aligned}$$

Π_A is maximized at $r_A = s_A$. For all situations $r_A = s_A$ maximizes the profit, so

$r_A^*(p_B, s_B) = s_A$. Similarly, $r_B^*(p_A, s_A) = s_B$. \square

Proof of Proposition 5.5.2:

- (1) Solve $p_A = \frac{1}{6}(3Emax(v, s_A) - Emax(v, s_B) + p_B + L + 3c_A)$, and $p_B = \frac{1}{6}(3Emax(v, s_B) - Emax(v, s_A) + p_A + L + 3c_B)$. In symmetric case, $p_A = p_B = \frac{1}{5}(2Emax(v, s) + L + 3c)$.

This will be an equilibrium if $p_A < \tilde{p}_A$. That is, $p_A < Emax(v, s) - \frac{L}{2} \Leftrightarrow L < \frac{6}{7}(Emax(v, s) - c) = \underline{L}^{PP}$.

- (2) If $p_A = p_A^{2*}$, then $p_A = \frac{1}{2}(Emax(v, s_A) + c_A)$. Similarly, $p_B = \frac{1}{2}(Emax(v, s_B) + c_B)$.

This point is equilibrium if $p_A > \tilde{p}_A$. That is, $p_A > Emax(v, s) - \frac{L}{2} \Leftrightarrow L > (Emax(v, s) - c) = \overline{L}^{PP}$.

(3) If $\underline{L}^{PP} < L < \overline{L}^{PP}$, then $p_A = \tilde{p}_A$. This will be an equilibrium if $\frac{\partial \Pi_A^1}{\partial p_A}(\tilde{p}_A) > 0$ and

$\frac{\partial \Pi_A^2}{\partial p_A}(\tilde{p}_A) < 0$, $2(Emax(v, s_A) - p_A) - \frac{3}{2}(p_A - c_A) > 0$, and $2(Emax(v, s_A) - p_A) -$

$2(p_A - c_A) < 0 \Leftrightarrow \frac{1}{2}(Emax(v, s) + c) < p_A < \frac{1}{7}(4Emax(v, s) + 3c)$.

□

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